International J. Soc. Sci. & Education
2012 Vol. 3 Issue 1, ISSN: 2223-4934 E and 2227-393X Print

How does the Use of Mathematical Symbols Influence Understanding of Mathematical Concepts by Secondary School Students?

By

Silvanos Chirume
Department of Mathematics and Statistics, Faculty of Science and Technology
Zimbabwe Open University, Midlands Region.
PO Box 1810 Gweru, Zimbabwe [skchirume@gmail.com]

Abstract

If students understand and properly communicate using math symbols and notation, their achievement in math might improve (Rubenstein & Thompson, 2001). This study investigated how the use of mathematical symbols influences understanding of math concepts by secondary school students in Shurugwi District (Zimbabwe). Convenience sampling (for the district), simple random sampling (for the schools), judgemental sampling (for the teachers) and stratified random sampling with proportional allocation (for the students) were used. The sample included six schools, 120 "O" level students, 27 "A" level students and 7 teachers of those students. The "O" level students completed questionnaires and were interviewed as a group at each of the 6 selected schools. "A" level students wrote a diagnostic test and were later interviewed as a group while the teachers were individually interviewed. It was found that most students fail to understand or interpret the meaning of math symbols due to the way they are taught to read, pronounce and use them. This misuse (and also abuse) of symbols may considerably hinder formation, understanding and communication of concepts and might affect achievement; the final outcome desired. Teachers are therefore sensitized on appropriate strategies to take to overcome students' difficulties on the use of symbols. The strategies include informed choice of the main classroom textbook to use, integration of math with other subjects and a firmer grasp of the subject matter and its pedagogy.

Keywords: *Symbols, symbolism, notation, misuse, abuse, influence, maths concepts*

1. Introduction

In this modern technological world Mathematics plays a greater role than ordinary language in trying to find solutions to everyday problems. Coding and decoding information, shortening sentences and representing and analyzing data are all processes where mathematical symbols are used. Mathematics is also itself a language with an internationally recognized syntax and vocabulary (Esty, 2011). However, the way in which Mathematics exploits the spatial features of its symbolisms and develops manipulations of symbolic expression is a special property not shared with ordinary languages such as English or Shona.

While teaching in Shurugwi District, the researcher found out that most students failed to grasp maths skills and concepts. The reason for that failure could have been that the symbols which they encountered were unfamiliar, confusing and sometimes contradictory. The researcher then became interested in finding out the truth about this conjecture, focusing on the topics on sets at "O" level and calculus at "A" level.

Contextual and Theoretical Frameworks

Some people might argue that all Mathematics deals with symbols and notation and that books and teachers always explain their use to students, hence there is no problem at all. This might not be true. Problems on the misuse (and also abuse) of mathematical symbols have always been there since man started counting. According to Kline (1972), Diophantus introduced symbolism before the 16^{th} century to replace special words, abbreviations, and number symbols which were a common style in the Renaissance. The pressure to introduce symbols in the 16th century came from the expanding scientific demands on Mathematics. However, many changes were made by accident. By the end of the 17th century the deliberate use of symbolism, not incidental or accidental, and the understanding and interpretation of it entered mathematics. The problem then, was that far too many symbols were introduced.

Since the problems of inventing, using and interpreting symbols have always been there, alongside other developments in mathematics, confusion and mistrust among mathematicians resulted in many symbols not being standardized. Kline (1972, pp. 260-262) describes the problems associated with this historical development and points out that, “The terms and notation varied a great deal; many symbols were derived from abbreviations.... But as far as one can judge, the introduction of letters for classes of numbers was accepted as a minor move in the development of symbolism."

Today the problem is no longer on inventing symbols, but on using, reading and interpreting them. Here are some of the problems and hypotheses on the use of mathematical symbols and how it influences understanding and mastery of concepts:

- According to Pimm (1987) the problem is that the symbols themselves are taken as the objects of mathematics rather than the ideas and processes which they represent.
- Pupils fail to interpret or understand the meaning of certain mathematical symbols due to the way by which they are taught to read those symbols.
- Pupils studying alone at home usually do not know how to read many mathematical symbols because they seldom hear them being spoken. For example, some sixth form pupils could read the first element of a matrix a_{11} as "a eleven" or "a subscript eleven" instead of "a one one".

A lot of research on how the use of mathematical symbols influence understanding of concepts has been carried out at the primary school level and probably very little at secondary school level. However, the general consensus is that the introduction of mathematical symbols presents difficulties and challenges beyond those presented by words alone (Kuster 2010, Lee 2004).

Marjoram (1974) cites a case where, out of a hundred eleven year old pupils, an objective arithmetic test full of symbols was given and all the pupils could not get even forty percent of the marks, but when that same test was conducted verbally only thirty pupils failed to score forty percent. He claims that most of these children could do the mathematics mentally and verbally but were "defeated" by the symbols. This defeat could be due to abuse or misuse of the symbols. If mathematical symbols are misused, pupils' understanding of concepts would be greatly retarded. Since mathematics is also a language, both words and symbols need to be used simultaneously. According to Marjoram (1974, p. 5), "Pure language served to take the Greeks a long way in their development of geometry, but of course they used diagrams."

Diagrams are Mathematical symbols, so it can be said that the Greeks also used symbols in geometry. So why do students fail to grasp even easy mathematical concepts? Earle (1977) argues that the problem lies on how symbols are used and perceived by the students. If a student cannot recognise and pronounce a symbol correctly, then he or she will have difficulties in using it. Earle (1977, p.6) gives $\sqrt{4}$ as an example and says, "If a reader is unable to pronounce $\sqrt{4}$ as `the square root of 4,' it stands to reason that he will have an inordinate amount of difficulty in mastering more sophisticated tasks involving those symbols." This is true but still the question: "To what extent does the use of mathematical symbols influence that mastering of sophisticated (or easy) tasks and how can we measure that effect?" can be posed.

Notable contributors to mathematics education such as Richard Skemp and Z.P Dienes have tried to answer some of these questions through their research. Skemp in Chapman (1972) gives us some light regarding the uses or functions and characteristics of symbols. He lists the functions as:

(1) For communicating with one another
(2) For becoming aware of our mental processes
(3) For recording knowledge for oneself or for others.

Symbols are also there to shorten our work and save time when we are writing. The use and effect of a symbol is measured by its characteristics; among other things. In support of this view, Skemp in Chapman (1972, p. 203) says, ".... handing on of knowledge to contemporaries and successors is made possible very largely by the use of written or drawn symbols; and it is to me surprising that so little effort has yet been given to finding out what makes a good symbol." Skemp further suggests that it is easier to find good symbols than bad ones; by use of an intuitive choice. For example, ξ (for universal set) and $\aleph$ (the Hebrew aleph) give trouble when writing. Mathematical symbols should also be easily distinguishable. For example, ν (for Roman vee) and υ (for Greek upsilon) look very much alike but could stand for different things. Kuster (2010, p. 221) gives what he calls ten quality criteria for mathematical symbols. He believes that good symbols should be readable, clear and simple, needed, international (or derived from Latin), mnemonic, writable, pronounceable, similar and consistent, distinct and unambiguous, adaptable and available.

Chapman (1972) recognises the importance of symbols and suggests that mathematical applications can be effective if the calculations are summarised and structured by means of good notations. The proper use of (good) notations can be understood and put into effect if one knows the features or principles employed in maths symbolism. These are colour, order, position, relative size, orientation and repetition. Some examples of these principles are,

(i) <u>ORDER:</u> The digits 1 and 7 are ordered differently in 17 and 71.
(ii) <u>POSITION:</u> 23 and 2^3, x^3 and x_3 are different.
(iii) <u>RELATIVE SIZE:</u> \$1.625 is different from \$1.62'^{5}. The last one diminishes the importance of the final digit.
(iv) <u>ORIENTATION:</u> That is, what makes a particular symbol distinctive. For example, d and p, 9 and 6, u and n are distinct.
(v) <u>REPETITION:</u> 88, xx (now x^2), $f^{/}(x)$, $f^{//}(x)$, $f^{///}(x)$
(vi) <u>COLOUR</u> In special circumstances (e.g. teaching slow learners), negative numbers can be written in green, positive numbers in red and unsigned numbers in black. These principles agree with Kuster's (2010) quality criteria for mathematical symbols.

Dienes (1963) says that when one learns mathematics, it is customary to "learn" the mathematics, talk about it and then write it in one lesson but it is doubtful whether pupils can distinguish between the maths learned and its symbols. He further suggests that the Maths symbols are not symbols, properly speaking, because symbols stand for something which they themselves are not; hence the misuse (or abuse) of symbols in maths teaching. Probably no one, however, has done extensive research on the use of these symbols. Dienes (1963, pp. 125-126) notes that, "A great deal of research needs to be done on the process of transition from the non-symbolic to the symbolic type of thinking."

Dienes conducted several studies on pupils' conception of cubes, squares and fractions using real and concrete objects. Pupils were asked to construct some shapes using different pieces labelled **NxNxN**, **NxNx1**, **Nx1x1** and **1x1x1** and then compare their shapes with, say, one symmetrical piece labelled $\mathbf{N^2+4N+4}$. The findings were that pupils failed to make some generalized formulation such as, say, "same number of rows as there are unit cubes in a row" and shorten that using symbols to **NxN** or $\mathbf{N^2}$. Dienes noted that pupils resisted a rapid march into symbolization if the necessary abstraction had not been achieved.

In Land (1963), Dienes argues that understanding of Maths concepts is increased or hindered by how we use symbols together with how we design the process of learning. He proposes three stages of the learning process, namely,

(i) An abstraction process

(ii) A symbolization process and
(iii) The use of symbols.

There should be feedback into the original experiences at every stage. It is the researcher's belief that most teachers do not bother to follow these stages since, in most cases, they may know very little about symbols.

From Dienes' point of view, there are two problems connected with symbols, namely, the process of symbolization and the use of symbols once symbolization has been established. He further asserts that, "We do not know whether the use of symbols is a help or hindrance in the formation of concepts." (Land, 1963, p. 54). Research still needs to be conducted on that.

Another aspect in line with the use of symbols is that of language. Quite a lot of research has been conducted on the use of language and its influence on student performance in Mathematics (Lee 1997, Flouris, Calogiannakis-Hourdakis, Spiridakis & Campbell 1994, Esty 2011). General findings according to Bell (1983) are that in a teacher-directed class, pupils will learn to use the teacher's language (however uncomprehendingly) whereas in a child-centred class unorthodox language presents the teacher with additional teaching problems. Rothery in Bell (1983) lays the blame on language together with the readability of Mathematical textbooks. He argues that exposition of concepts and methods, including explanations of vocabulary, rules and *notation* should be very clear in all Maths textbooks. The notation should not be abused (Esty 2011, Abuse of Notation: Wikipedia, 2010).

Purpose of the Study

The purpose of the study is to investigate the influence of mathematical symbols and their use on the way secondary school students grasp mathematical concepts. It is also intended to sensitize secondary school teachers to problems, or challenges, that students often have with mathematical symbols and to suggest instructional strategies that can reduce such difficulties since using symbols fluently and correctly is a necessary condition for overall mathematics achievement (Rubenstein & Thompson, 2001).

Data analysis may also provide some insight into these related questions.

(a) What is the best time for introducing symbols at secondary school?
(b) Should "symbols" be taught as a single topic or continuously together with other topics or subjects?
(c) Do many symbols representing the same concept hinder or aid learning of that concept if they are introduced?

2. Materials and Methods

Research Design

The descriptive survey methodology was used to find out how the use (or abuse) of mathematical symbols influences students' understanding of maths concepts. However, descriptive surveys are not very informative research designs because, "Descriptive surveys basically inquire into the status quo; they attempt to measure what exists without questioning why it exists." (Ary, Jacobs & Razavieh 1985, p. 337). To overcome this shortfall, interviews in the form of focus group discussions or "oral tests" with students were held. Students were allowed to discuss other difficulties they encountered in using maths symbols in an atmosphere of freedom of expression and one that would ensure that they were also free to criticize their teachers or their textbooks and report back the main points. These points were later discussed between the researcher and maths teachers of those students.

Sample and Sampling Procedure

Shurugwi District in the Midlands Province was conveniently chosen simply because the researcher had learnt and taught there and that is where the problems associated with mathematical symbols had been encountered. It is assumed that similar problems exist in other districts as well. With the help of the Shurugwi District Staffing Officer (DSO), all the secondary schools in the district were listed in alphabetical order. Then, using the random numbers on page 57 of Nelson (1980), the first six, two digit numbers less than twenty along the rows were selected. The number six, though small, was chosen because the researcher wanted to use a multi-method approach and to spend more time on discussions and diagnostic tests rather than on questionnaire administration. The secondary schools on the list corresponding to these 6 numbers then automatically became the sample schools. There was one school in the sample that had both "O" and "A" level students. The respondents were "O" and "A" level Mathematics pupils and their respective maths teachers and the assumption was that each level had covered the chapters on sets and calculus respectively. This assumption was later verified to be a true statement.

Since the study was about students' understanding of maths concepts and use of maths symbols, an unbiased study sample had to be chosen by controlling for the ability factor. Thus, Heads of Departments (HOD's) at the chosen schools were requested to assist the researcher to select twenty students at each "O" level school; six students of best performance, eight of average performance and six of poorest performance in Mathematics. The school with both "O" and "A" level classes had one 'A" level class of 27 maths students and these were also included in the sample. Seven mathematics teachers of the selected students (one from each "O" level school and one from the "A" level math class) were also included as respondents in the study. If there were at least two math teachers at a school, the HOD was tasked to release one for the interview. So, altogether the sample included 147 students comprising 120 "O" level and 27 "A" level as well as 7 maths teachers.

Instruments and Data Collection Procedure

Questionnaires were given to the selected "O" level students to complete at their own pace. There were 17 questions involving both open and closed ended statements on sets and other mathematical symbols. The reason for focusing on sets was that, basically, all mathematical structures can be explained in set-theoretic terms and the students who had been taught by the researcher before had difficulties in grasping and using the set symbols.

At one of the schools a discussion in the form of a diagnostic or "oral test" session with a class of 27 "A" level Maths students was held. These students were asked questions on differentiation and functions. Below are some of the questions:

(1) What is the distinction between $\frac{dy}{dx}, \frac{\delta y}{\delta x}$ *and* $\frac{\Delta y}{\Delta x}$?

(2) If f is a function, how would you read $f^{/}$? Select the correct answer(s).

A f feet
B f prime
C f minutes (of arc)
D first derivative of f
E second parameter of f.

(3) f is a function, a is any constant. We read f^{-1} as "f to the minus one" and a^{-1} also as "a to the minus one." But $a^{-1} = \frac{1}{a}$ whereas $f^{-1} \neq \frac{1}{f}$. Why is this so?

(4) $dy = f^{/}(x)dx$ and similarly $\delta y = f^{/}(x)\delta x$. True or false?

(5) From the following differentiation symbols select the one(s) which is(are) easy to write and easy to use.
(a) (i) Dy (ii) dy/dx (iii) $y^{/}$ (iv) $f^{/}(x)$
(b) (i) D^2y (ii) d^2y/dx^2 (iii) $y^{//}$ (iv) $f^{//}(x)$.

These discussions and diagnostic tests were enthusiastically carried out by students and they participated lively. The data collected from the questionnaires and the discussions revealed how students viewed their teachers, textbooks and mathematics itself. Unfortunately, the researcher did not get quite a lot of information from the teachers themselves, most of whom gave excuses of being busy.

3. Results and Discussion

Questionnaire Data

The questionnaire had seventeen questions and a total of 120 "O" level students from the sampled schools responded to them. Below is the question by question analysis of the results.

The first question was, "Have you had difficulties in reading and pronouncing mathematical symbols?" The responses and corresponding frequencies were as follows:-

A. Quite a lot ....40%(48) B. Few difficulties .... 26.7%(32)
C. Very few difficulties ....25%(30) D. No difficulties at all ...8.3%(10)

These results indicate that the majority of students (about 66.7%) had difficulties in reading and pronouncing mathematical symbols.

Question 2 was about the difficulties that students had in using symbols in sets. Interestingly, there were two (more or less equal) groups with contrasting views. Of all the students, 33.3% indicated that they had quite a lot of difficulties whereas 30% indicated that they had very few difficulties. These are the responses and frequencies:-

A. Quite a lot of difficulties ...33.3%(40) B. Few difficulties ....13.3%(16)
C. Very few difficulties ....30%(36) D. No difficulties at all ....23.3%(28)

Hence, it seems that on average, students did not have problems in using symbols in sets.

Question 3 was on what could be the easiest way of defining a set. The alternative responses and the frequencies were,
A. By listing its elements ....73.3%(88) B. By describing the elements ....26.7%(32)
C. Any other way (specify) ....0%(0)
Here language problems are revealed because one can conclude that most students wanted to "list" and not to "describe" things.

In question 4 there were only two alternative responses A:YES and B:NO to the given statement that many students find sets difficult to understand because some symbols which are used look alike but mean different things. The responses were 61.7% (74) for A and 38.3% (46) for B. Thus the majority agreed with the statement and this tallies with the researcher’s assumptions and also with findings of Rubenstein and Thompson (2001).

Question 5 was rather tricky. Symbols for the improper subset, less than or equal to, element of, and universal set were given and students were told that the symbols looked alike. Of the students, 1.7% (2) strongly agreed, 1.7%(2) agreed, 21.6%(26) were undecided, 41.7%(50) disagreed and 33.3%(40) strongly disagreed with the given statement. This might mean that students were quite familiar with the "structure" of the symbols. However, this does not mean that they knew the meanings of those symbols.

Question 6 tested students on the distinction between "subset of" and "contains". The question was:- Does $B \subset A$ and $A \supset B$ mean (a) exactly the same (b) almost the same or (c) opposites? Here 73.3%(88) chose (a), 20%(24) chose (b) and 6.7%(8) chose (c). The results indicate that students couldn't distinguish between $\subset$ and $\supset$.

Question 7: Answer true (T) or false (F) for (a) $B^{/}=B^{c}$ and (b) $(B^{/})^{/}=B$. The responses for (a) were True 43.3%(52), False 56.7%(68) and for (b) True 20%(24), False 80%(96). These results reveal that most students hadn't grasped the concept of "complement" firmly.

In question 8 the following symbols were given: $\supset, \cup, \subset, \cap$, and students were told that it is just the same symbol rotated four times through an angle of 90^{o} clockwise; so it stands for the same thing. The results were that 5%(6) strongly agreed, 5%(6) agreed, 26.7%(32) were undecided, 43.3%(52) disagreed and 20%(24) strongly disagreed. It can be concluded that at "O" level, there are students (10%) who still think that a symbol must retain its meaning even if it is rotated in such a way that it looks different from its original structure. They probably do not know that rotation (or orientation) changes the meaning of the symbol completely.

The symbols **E** and ξ representing the entire set were given in question 9 and students were asked to choose the symbol they preferred to use and to state why. Fifty five percent (66) chose **E** and 45%(54) chose the later symbol. Those who chose **E** gave reasons that it was "easy to write," or "more understandable" or that "the other one is complicated". Those who preferred the latter symbol gave reasons that **E** could not be a better one since it means "an element of," thus confusing it with $\in$.

The meaning of n(**E**) was tested in question 10. There is actually a difference between a(x) which means a times x in algebra and n(**E**) which does <u>not</u> mean n times **E** in sets. So, for the statement that n(**E**) means n times **E**, 28.3%(34) chose A:TRUE, 65%(78) chose B:FALSE and 6.7%(8) of the students chose C:UNDECIDED. From these results one can conclude that most students understood the meaning of n(**E**).

In question 11, the main idea was on using different symbols to represent an empty set. There were three alternatives, namely, (a) an empty circle (b) B={ } and (*c*)*B*=Æ. It is interesting to note that 66.7% (80) of the students thought that response (c) was the best to stand for "set B is empty" while 33.3%(40) chose B={ } and 0%(0) preferred just an empty circle with a letter outside the circumference.

In question 12, a circle, B, was drawn and inside it were two elements, a small circle (or zero) and the letter A outside it but near the circumference. There were four alternatives as to what the diagram might represent or mean, namely,
(a) Set B contains two elements; the letter A and the number 0 (b) Set A is a subset of set B
(c) Set B contains two elements; the letter A and a small circle (d) Any other conclusion (specify).
All the responses a, b, and c would be correct. Thirty five percent (42) chose (a), 18.33%(22) chose (b) and 45.83%(55) chose (c). Only 0.83%(1) chose (d) but did not give any reasons.
Maybe he or she did not know the meaning of "specify".

Item 13 was the statement that students fail mathematics because there are too many symbols to learn and understand. The respective responses and frequencies were as follows:-
(a) Strongly agree.... 33.3%(40) (b) Agree ... 30%(36) (c) Undecided ... 0%(0)
(e) Disagree ....10%(12) (d) Strongly disagree.....26.7%(32).
These results indicate that most students strongly agreed that failure of mathematics is caused by too many symbols which need to be learnt and understood.

Question 14 included a universal set E with two intersecting subsets A and B. There were some geometric shapes as elements in each set. Students were asked to describe fully and in words the set E. Language difficulties in mathematics were revealed in the responses to this question. About 78.3%(94) of the students could not describe the given set E fully and correctly. Those who did better, 21.7%(26), did so but still in poor English.

Question 15 required students to name some topics in maths where they have had difficulties in understanding the symbols given. It was observed that sets, probability, inequalities, variation and trigonometrical ratios were some of the topics mentioned by the majority (93.3%) of the students. This confirms the researcher's hypothesis that most "O" level students have problems understanding mathematical symbols and these problems might reduce performance in mathematical problem solving (Luna & Fuscablo, 2002).

Question 16 asked for those particular symbols. Some of the mentioned ones were symbols for less than or equal to, subset and "contains", union and intersection, alpha, $A^{/}$, A^{-1}, $(AUB)^{/}$ and the universal set. Since these symbols have to be explained in English, students may also have a "double jeopardy" if English is their second language (Garegae, 2011).

The reasons for not being able to grasp the meanings of the symbols and to use them appropriately were indicated in question 17. Thirty-three percent (40) of the students blamed the teachers who "didn't explain what the symbols meant", 5%(6) gave various reasons such as "no enough time to study", "no skilled teachers" and "there are too many symbols in maths" while 62%(74) blamed the "shallow textbooks" and themselves for their failure to use maths symbols appropriately.

Interview Data

From the discussions carried out with the "O" level students at the chosen schools, the majority of them said that they had not experienced any difficulties in using mathematical symbols and understanding the required concepts. However, when further interviewed and given a short test involving maths symbols, they failed to solve the given problems. This indicated that in general, students had a lot of difficulties but maybe were shy to say so. Maybe they feared that they would be labelled "dull" or "ignorant" which was not the case.

Colour is seldom used in printed mathematical notation because it is difficult to replicate. Nevertheless, 74%(20) of the "A" level students at one school said they preferred the use of different colours for negative numbers, positive numbers and unsigned numbers to that of no colour at all. The researcher concurred with them only for the reason that when one uses colour, there is no confusion arising from different uses of the same sign; for example, reading -2 as either "subtract two" or "negative two." The others , 26% (7), had nothing to contribute.

Diagnostic Test

A short diagnostic test involving symbols used in differentiation and functions was given to the "A" level students at one of the schools. From the results of that short test, it was found that 92.6%(25) of the students were familiar with dy/dx, $y^{/}$ and $f^{/}(x)$ but 100%(27) were unfamiliar with Df(x). However, the distinction between $\frac{dy}{dx}, \frac{\delta y}{\delta x}$ and $\frac{\Delta y}{\Delta x}$ was not firmly understood. Thus they said if $dy=f^{/}(x)dx$ was true, $\delta y=f^{/}(x)\delta x$ was also true; which in fact, is not necessarily true. Nobody said dy/dx is a fraction except 3.7%(one boy) and nobody agreed that $f^{-1} = 1/f$ if f is a function.

Most "A" level students, 85.2%(23) said that they knew the symbols and their meanings but could not express themselves clearly and logically. They had language difficulties. In response to one question asked, most students were of the opinion that $y^{/}$ and $y^{//}$ were easy to write while $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ were easy to understand.

General Findings

From the questionnaires that were administered and the interviews and diagnostic tests carried out with both "O" and "A" level students, it was found that other difficulties in learning maths arise not from the vocabulary of mathematical writings but from their linguistic "structure." Thus language plays a key role in the use and mastery of mathematical symbols. Pimm (1987, p.148) has similar views and further suggests that, "It clearly depends on the language of the reader, and on occasion,...,there will be a conflict between the conventional letter employed in Mathematics and an unwillingness (on grounds of easing the memory load) to employ that particular letter."

Most students responded to questions 4 and 13 in the questionnaire by accepting that there were too many symbols in maths some of which varied a lot in meaning and/or structure. This is in line with Preston (1978) who criticises the great variation in symbols and terminology used in maths. But there are situations in which expressing mathematical ideas in more than one way may be beneficial. Preston (1978, p. 287) writes, "Situations in which the use of more than one system of notation may be either helpful or confusing, for example, could be quoted." Unfortunately, the author does not quote those situations.

Discussions with Teachers

The results got from the discussions with the teachers were quite interesting. About 57%(4) of them echoed that positioning symbols plays a very important part in maths learning. They said they had difficulties in explaining, for example, that a^{b+c}, ab+c and a^b+c mean different things although they are the same symbols in the left to right order. Other teachers, 43%(3), had problems when teaching with the aid of diagrams, especially those of geometric figures. They indicated that students who see the symbol for a rectangle think that it is the actual object of study and then start to measure it with a ruler and a protractor. Teachers pointed out that even the "equals" sign (=) is usually misunderstood and abused. For example, when solving linear equations, the proper use of the equals sign is neglected. We often see this:-

$$\begin{array}{l} 3x-5=x+9 \\ =3x-x= \quad 9+5 \\ =2x \;\; = 14 \\ \;\;2 \quad\;\; 2 \\ \underline{= x \;\; = 7} \end{array}$$

Notice that the division line (in step 3) is left out. It is also interesting to note that when simplifying algebraic expressions, students "forget" to properly use the equals sign. For example,

<u>Simplify</u> (2a+b)(3a-b)

<u>Solution</u> (2a+b)(3a-b)

$$\begin{array}{l} =2a(3a-b)=b(3a-b) \\ 6a^2-2ab+3ab-b^2 \\ \underline{6a^2+ab-b^2}\,. \end{array}$$

The discussions also motivated teachers to be aware of their role in helping students to be reflective thinkers. According to The Progressive Education Association (1940, p. 215),

> Teachers of Mathematics use symbols skilfully in their own subject, but few of them think about their use of symbols.... Teachers seldom realize that they can help students to develop their own powers of reflective thinking through a more conscious attention to symbols in the Mathematics classroom.

Finally, through these discussions and personal observations, it was found that most textbooks just use difficult words and symbols without giving thorough explanations and clear examples. Earle (1977) also argues that there is an element of redundancy in all text material which makes it impossible for students to perceive every symbol. This needs to be explored further.

4. Conclusions and Recommendations

This study has investigated how the use of set theory and calculus symbols, and others, influence understanding of concepts by secondary school students. It has also sensitized teachers on appropriate strategies to take to overcome students' difficulties on the use of symbols. From the findings of the study, there is evidence that most students fail to interpret or understand the meaning of mathematical symbols due to the way by which they are taught to read, pronounce and use them. This misuse (and abuse) of symbols considerably hinder formation, understanding and communication of concepts to a great deal and might affect the final achievement.

This study has also concluded that students fail to grasp mathematical concepts because they take the symbols themselves as the objects of mathematics rather than the ideas and processes which they represent. According to the results from the questionnaires, the blame lies on the textbooks and the teachers. Teachers seldom explain the meanings and proper uses of the symbols while textbooks change the symbols too often and don't bother to give historical background information about those symbols. Students fail because teachers introduce new words or symbols when the given situation can be handled in terms of words and symbols already known.

Drawing from the problems and difficulties mentioned above and their possible causes, the following recommendations for an effective and proper use of symbols that would lead to a firmer grasp of mathematical concepts are given:

- On textbooks, it is the duty of everybody concerned with maths education to:-

(a) improve the text
(b) improve the teacher's use of the text and
(c) improve the reading ability of the reader.

- When recommending textbooks, teachers should select those that provide short historical accounts of mathematics, the mathematicians involved, the dates and the symbols they used. The textbooks should explain why certain symbols were dropped and yet others were accepted internationally. If such textbooks are not available, historians, mathematicians and educators can work together to produce them. The teachers and textbooks should avoid continuous use of symbols that are complicated and difficult to understand, difficult to write (sometimes needing a computer) and confusing and contradictory. It is helpful for teachers to make sure that students understand the meanings of the symbols even though they allow the students to manipulate such symbols mechanically.

- The first lesson about symbols for secondary school students should emphasize strongly the fact that symbols are instruments or tools of thought. Another lesson should focus on the fact that a given symbol may often serve a variety of purposes. For example, the symbol "e" is used as a base in logarithms, as the identity element in abstract algebra ($e * x = x * e = x$) and as the coefficient of restitution in mechanics. Thus, it is important to study the setting and context in which the symbol is used.

- The teacher should also be well versed in mathematics in general and in the use of maths symbols in particular. In the classroom the good teacher can introduce games involving the use of symbols, constantly referring to the school library section on maths games. Using overhead or micro-soft power point slide projectors, any chosen student can read aloud the written symbol, tells the topic (area) where the symbol is used and then spells the "name" of the symbol while others record time. These activities could be done at Z.J.C level (form 2) and below as games and at "O" level as remedial work for slow learners. The teacher should also display on the wall several charts which carry different symbols, what they mean and where they are used. Students can then practise pronouncing them and using them in sentences as suggested by Rubenstein and Thompson (2001).

- Mathematical symbolism should be integrated with other topics or subjects at the beginning of every course and be sustained at all levels of students' learning (Luna & Fuscablo, 2002). Teachers should not take symbols for granted and should not by-pass them in their discussions (Chapman (1972, p. 38).

In conclusion, symbols should be used only after a satisfactory explanation of their meanings has been given, otherwise they should be accepted worldwide. Errors in reading and pronouncing symbols should be identified and remedied. The meaning of each symbol or each symbol string should be razor sharp and unambiguous. That way, mathematical concepts can be firmly understood and grasped. Teachers are hereby challenged to use suggestions and recommendations given in this study and to carry out classroom action research on the use of symbols and their impact on student achievement.

Acknowledgements
I would like to express my heartfelt gratitude to the following people without whose help and cooperation the research article would not have been a success.

(1) DR. A.K Babugura - Who inspired and encouraged me to continue researching on the topic of symbols.

(2) DR. D.K Mtetwa - Who read the paper in its early stages and gave valuable suggestions.

(3) Teachers and students of secondary schools in Shurugwi District who cooperated by allowing me to interview and test them. Their names, unfortunately, cannot be disclosed because the information they provided is to be treated confidentially.

(4) The Midlands Regional Director and Shurugwi D.S.O for allowing me to research in their schools and providing me with the schools list.

References

Ary, D., Jacobs, L.C. & Razavieh, A., (1985). *Introduction to Research in Education*. New York: Holt, Rinehart and Winston.

Bell, A.W., Costello, J. & Kuchemann, D.E., (1983). *A Review of Research in Mathematical Education; Part A: Research on Learning and Teaching*. Windsor, Berks: NFER-Nelson Publishing Coy Ltd.

Chapman, L.R., ed. (1972). *The Process of Learning Mathematics*. Oxford: Pergamon Press Ltd.

Dienes, Z.P., (1963). *An Experimental Study of Mathematics Learning*. London: Hutchison and Co. Ltd.

Earle, R.A., (1977). *Teaching Reading and Mathematics*. Delaware: International Reading Association.

Esty, W., (2011). *The Language of Mathematics*. Retrieved on 06/12/11 from

http://www.augustusmath.hypermath.net

Flouris, G., Calogiannakis-Hourdakis, P., Spiridakis, J., & Campbell, J.R., (1994). Tradition and Socio-Economic Status are Greek Keys to Academic Success. *International Journal of Educational Research* (21), 7, pp. 705-712.

Garegae, K.G., (2011). *Language in Mathematics Education. A double Jeopardy for Second Language Learners*. Retrieved on 15/12/11 from

http://www.tsg.icme11.org/document/get/120

Kline, M., (1972). *Mathematical Thought from Ancient to Modern Times*. New York: Oxford University Press.

Kuster, J., (2010). Math Never Seen. *TUGboat* (31), 2, pp. 221-229.

Land, F.W., (1963). *New Approaches to Mathematics Teaching*. London: Macmillan and Co. Ltd.

Lee, C., (1997). Teachers Can do Research. *Mathematics Teaching* (158), pp. 8-11.

Lee, X., (2004). *The Problems of Traditional Math Notation*. Retrieved 06/12/11 from http://www.xahlee.org/cmaci/notation/trad_math_notation.html

Luna, C.A. & Fuscablo, L.G., (2002). Enhancement of Student Problem Solving Performance through Mathematical Symbolism. *Proceedings of ICME10-Topic Study Group 9*, pp. 1-10.

Marjoram, D.T.E., (1974). *Teaching Mathematics*. London: Heinemann Educational Books.

Nelson, R.D., (1980). *The penguin book of mathematical and statistical tables*. Harmondsworth: Penguin Books Ltd.

Pimm, D., (1987). *Speaking Mathematically*. London: Routledge and Kegan Paul.

Preston, M., (1978). The Language of Early Mathematical Experience. *Mathematics in School* (7), 4, pp. 31-32.

Progressive Education Association, (1940). *Mathematics in General Education*. New York: Appleton-Century Crofts.

Rubenstein, R.N. & Thompson, D.R., (2001). Learning Mathematical Symbolism: Challenges and Instructional Strategies. *Mathematics Teacher* (94), 4, Reston, VA: NCTM.

Wikipedia, (2010). *Abuse of Notation*. Retrieved on 06/01/12 from

http://www.en.wikipedia.org/wiki/Abuse_of_notation