International J. Soc. Sci. & Education
2015 Vol.5 Issue 4, ISSN: 2223-4934 E and 2227-393X Print

Conceptions and Concept Images of Prospective Mathematics Teachers in a Teacher Training Program Regarding Basic Mathematical Concepts

By

Davut Köğce
Mathematics Education, Department of Elementary Education, Faculty of Education,
Nigde University, Nigde, **Turkey.**

Abstract

This study identified the extent to which prospective mathematics teachers attending a teacher training program were able todescribe certain basic mathematical concepts, namely the set of real numbers, the set of RxR, rational numbers, repeating decimals, absolute value, equations, inequations, relations and functions. It was designed as a case study. The sample was composed of 58 prospective mathematics teachers who attended a teacher training program at Nigde University's Faculty of Education during the spring of the 2014-2015 academic year and took the courses School Experience and Special Teaching Methods as part of this program. The data were collected using a survey with 16 open-ended questions about the mathematical concepts. The data were analyzed descriptively. The results showed that the prospective teachers (PT) had difficulty describing the mathematical concepts properly and had scientifically inaccurate concept images. The presentation of the results is followed by recommendations.

Keywords: *Concept image, Mathematics Instruction, Basic Mathematical Concepts, Prospective Mathematics Teachers*

1. Introduction

The fact that students can solve a mathematical problem accurately does not necessarily mean that they have fully understood or can describe the relevant mathematical concept (İşleyen & Işık 2005). This is because there are many mathematical operationsthat students can perform without being able to explain them. Bingolbali and Özmantar (2010) maintain that students' conceptual development is hindered when teachers use slide presentations and simple lecturing with a teacher-centered approach or when they make their students memorize formulas. To avoid this and thus provide effective mathematics instruction, emphasis on conceptual and procedural knowledge should be balanced (Zembat, Özmantar, Bingölbali, Şandır & Delice, 2013; Birgin & Gürbüz 2009; Soylu & Aydın, 2006). Therefore, the definitions of concepts should be seen as very important in mathematics instruction (Aydın & Soylu, 2006).

According to a model proposed by Tall and Vinner (1981), students' understanding of a concept can be analyzed using two components: concept definition and concept image. A concept definition is a formal definition accepted by the scientific community. For example, formal definitions of mathematical concepts are general definitions that are recognized by mathematicians and the mathematical community. A concept image is an informal description that includes all mental pictures emerging in mind, shaped by impressions, experiences or prior knowledge. Images are personal, varying from one person to another and involve pictures of the concept, insights into its properties and words or phrases that connote it (Bingolbali & Monaghan, 2008). Since a concept image is informal and generated in mind through experience, either consciously or unconsciously, it may involve misconceptions. In addition, a concept image does not have to be appropriate for or consistent with the concept. Instead, students can have contradictory views on the concept without being aware of this (Rösken & Rolka, 2007). Studies of the model by Tall and Vinner (1981) and Vinner (1991) suggest that students tend to use concept images rather than established concept definitions when they attempt to construct a new concept.

A number of studies in the literature have focused on the identification of the misconceptions of elementary, secondary and high school students, as well as prospective mathematics teachers, about certain basic mathematical concepts (Akyüz & Hangül, 2014; Baki & Aydın Güç, 2014; Özkaya & İşleyen, 2012; Kaplan, İşleyen & Öztürk, 2011; Yenilmez & Avcu, 2009; Moralı, Köroğlu & Çelik, 2004) and on conceptual and procedural learning (Birgin & Gürbüz, 2009; Soylu & Aydın, 2006; Baki & Kartal, 2002). However, there is limited research on the definitions of mathematical concepts (Dane & Başkurt, 2012; Aydın & Köğce, 2008). The available research reports that students have many misconceptions and their conceptual knowledge and procedural knowledge are not well-balanced. It also finds that they have difficulty describing concepts and fail to make connections between concepts.

A number of studies have been conducted, both in Turkey and around the world, using the model proposed by Tall and Vinner (1981). Most of them have focused on the rectangle as a geometric concept (Türnüklü Gündoğdu-Alaylı & Akkaş, 2013; Erşen-Bahar & Karakuş, 2013; Duatepe-Paksu, Musan, İymen & Pakmak, 2012; Bozkurt & Koç, 2012; Ergün, 2010; Akuysal, 2007; Fujita & Jones, 2007; Aktaş, 2005; Monaghan, 2000; De Villiers, 1994; Wilson, 1990), while others have examined trigonometric concepts (Akkoç, 2008; Topçu, Kertil, Akkoç, Yılmaz & Önder, 2006), irrational numbers (Kara & Delice, 2012) and limits (Duru, 2011). The former group of studies reported that students often have difficulty describing shapes and arranging them in hierarchical order. In addition, their personal descriptions of rectangular shapes, especially trapezoids, are characterized by inaccurate concept images. The studies of trigonometric concepts reported that none of the students were able to describe a radian and their concept images were influenced by their concept images' levels. In the study of irrational numbers, the students described an irrational number as a number that is not rational and its symbolic expression as a square root. The students were also unsure about the irrationality of numbers such as "π" and "e." In the study of the concept of limit, the prospective teachers' concept images were shaped by the idea that values assigned from the right and left bring about equality, and some misconceptions emerged as they were trying to generate concept images of limits.

Today, most teachers believe that success in mathematics means the ability to use formulas, rules and methods simultaneously and properly and to make accurate calculations (Soylu & Aydın, 2006). Therefore, most students are not aware that their attempts to solve mathematical problems use operations that are actually based on mathematical concepts, nor do they know what mathematics means. Students see mathematics learning as performing operations using meaningless symbols and formulas and try to memorize mathematics rather than learn it (Soylu & Aydın, 2006; Oaks 1990). When elementary school subjects are not taught in a way that will enable students to fully comprehend them, the result is misconceptions and incomplete conceptions, which persist in high school and even university education (Yılmaz & Yenilmez, 2008).

One reason why students tend to get poor scores on national and international examinations (e.g. YGS, YLY and TIMS) may be that teachers fail to explain mathematical concepts effectively and let them struggle with incomplete concept definitions, thereby indirectly leading them to memorization. This assumption is supported by many cases that have been observed in education. For example, the Council of Higher Education in Turkey decided in 2014 that graduates of faculties of science and letters would be entitled to become a teacher if they have a teacher training certificate. According to prospective mathematics teachers who attend teacher training programs and visit schools as part of the course School Experience, the approaches of their teachers are not consistent with those stipulated in the mathematics curriculum since they simply provide rules and formulas and allocate much more time to problems that call for procedural knowledge. In addition, prospective mathematics teachers in pedagogical training programs fail to describe basic mathematical concepts properly and to give incomplete descriptions when they are asked to make presentations on certain topics as part of the course Special Teaching Methods. This is despite the fact that students or graduates of faculties of science and letters are expected to have no difficulty describing mathematical concepts since they are taught comprehensive theoretical courses throughout their university education and their pedagogical content knowledge should go well beyond the mathematics curriculum for secondary schools or high schools.

This study is motivated by the fact that the literature on mathematical concepts does not contain any studies on students of mathematics attending a teacher training program, that students get poor scores on national and international mathematics examinations and that prospective mathematics teachers in pedagogical training programs are made to adopt a teaching style based on memorization and procedural knowledge rather than conceptual knowledge. The purpose of this study is to identify what conceptions and concept images prospective mathematics teachers attending teacher training programs have about certain basic mathematical concepts, namely the set of real numbers, the set of RxR, rational numbers, repeating decimals, absolute value, equations, inequations, relations and functions.
Accordingly, this study's problem statement is:

How do prospective mathematics teachers in teacher training programs perceive and describe certain basic mathematical concepts (the set of real numbers, the set of RxR, rational numbers, repeating decimals, absolute value, equations, inequations, relations and functions)?

2. Method

Study Model

This descriptive study was designed as a case study since it attempts to identify the extent to which prospective mathematics teachers in a teacher training program can describe certain mathematical concepts and their concept images of them. A case study allows researchers to perform an in-depth analysis of one aspect of a given problem in a short time. Although generalization is not the principal objective of such studies, it is still likely that their results will shed light on general situations (Çepni, 2007; Ekiz, 2003).

Participants

The sample was determined using purposive sampling. It was composed of 58 prospective mathematics teachers who attended a teacher training program at Nigde University's Faculty of Education during the spring of the 2014-2015 academic year and took the courses School Experience and Special Teaching Methods as part of this program. This sample was chosen because the students in the teacher training program in the previous semester had difficulties and problems with describing this study's basic mathematical concepts.

The Data Collection Instrument

The data were collected using a survey with 16 open-ended questions. The survey was designed to identify the extent to which the prospective teachers were able to describe basic mathematical concepts (Table 1).

Before the survey was designed, a comprehensive literature review was conducted. The first part of the survey consisted of eight structured open-ended questions that required the participants to offer descriptions of the basic mathematical concepts. The second part contained eight structured open-ended questions that required the participants to perform operations based on their descriptions. The open-ended structured questions enabled the participants to explain their answers and the thinking processes they used to find their answers (Gronlund & Linn, 1990). The survey form was submitted to two specialists in mathematics instruction and one specialist in mathematics. It was revised according to their recommendations.

Table 1. The questions in the data collection instrument

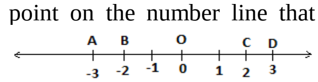
PART I-Questions about Concepts	
1. a) Describe the set of *IR* (real numbers) and **2.** b) *IRxIR* and explain the main differences between the two? Please give examples.	**5.**What is equation? Please describe. What makes an expression an equation?
3. What is rational number? Please describe.	6. What is inequation? Please describe. What makes an expression an inequation?
4. Is a repeating decimal a rational number? Why? Please explain your answer in detail.	**7.** What is relation? Please describe. What makes an expression a relation?
5. What is absolute value? Please describe.	**8.** What is function? Please describe. What makes an expression a function?
PART 2-Procedural Questions for the Concepts	
1.a) What set are the following numbers a member of: -100, $-\sqrt{2}$, -1, 0, $\frac{3}{5}$, $0.13\overline{2}$, $\pi$and 1000? Why? Please explain your answer. b)What sets are the following points a member of: *A(-3,7)* and *B(5,1), C(π,* $\sqrt{3}$)? Why? Please explain your answer.	**5.** Please circle the equations and explain why they are equations. a) $2x^2 - 3x + 2$ b) $y = 2x - 3$, c) $3x - 5 = 0$, d) x=0 e) $(x-2)^2 = x^2 - 4x + 4$, f)$2x - 2y < 4$
2. Please circle the rational numbers and explain why they are rational numbers. 0, -7, $\frac{3}{5}$, $0.3\overline{2}$, π, 0.21, $-\sqrt{2}$, $2\frac{2}{5}$, 3!	**6.** Determine the largest possible domain of $4 < x^2 < 25$. Please explain how you did so.
3. a) What is the solution to the following problem: $1{,}233 + 0.\overline{5}$ =?Please explain your answer. b) Please write an equation for $0.1\overline{2}$and explain your answer.	**7.** Which ones of the following are a relation <u>from A to B</u>, where $A = \{-3, -2, 0\}$ and $B = \{1, 2, 3\}$ are sets? Why? Please explain your answer. a) $\beta_1 = \{(-3,1),\ (2,-2)\}$ b) $\beta_2 = \{(-3,1),\ (-3,2), (-3,3)\}$ c) $\beta_3 = \{(-3,1)\}$ d) $\beta_4 = \{(-3,1),\ (0,3)\}$ e) $\beta_5 = \{-3, -2\ , 0,\ 1,\ 2,\ 3\}$
4. What is the solution to the following problem: $\lvert\lvert x - 3\rvert + (2x-2)\rvert$ =?, where$-1 < x < 3$? Please explain how you solved it.	**8.** Which ones of the following are a function <u>from A to B</u>, where $A = \{0, 2, 4\}$ and $B = \{1, 2,\ 3\}$ are sets? Why? Please explain your answer. a) $\{(0,1), (0,2), (2,1),\ (4,\ 1)\}$ b) $\{(1\ ,0\ ), (2,4), (2,2))\}$ c) $\{(0,1), (2,1),\ (4,\ 1)\}$ d) $\{(0,2), (2,1), (4,\ 1)\}$ e) $\{(0,1), (2,\ \ 3),\ (4,\ 2)\}$

Data Analysis

The literature was reviewed comprehensively, and a rubric was developed to evaluate the answers to the questions in the data collection instrument (Table 2).

Table 2. The rubric used for data analysis

Concepts	Definitions in the literature
1-a)Real Numbers (R)	• The set of real numbers is a combination of the sets of rational and irrational numbers. Each member of the set of real numbers is called a real number. The relation among the sets of numbers is as follows: N$\subset Z \subset Q \subset R$ (Balcı, 2012). • The set that contains rational (Q) and irrational (Q') numbers is called the set of real numbers, and its symbol is R. Based on this definition, $R = Q \cup Q'$and N⊂Z⊂Q⊂R (Komisyon, 2014; Akdeniz, Ünlü & Dönmez, 2012; Dernek, 2009). • A combination of the set of rational (Q) and irrational (Q') numbers is called the set of real numbers, and its symbol is R. $R = \mathrm{Q} \cup \mathrm{Q}'$ Each point on the number line stands for a real number. Real numbers and the points on the number line can be matched one-to-one. Real numbers cover the entire number line (Akpınar, 2004; Polatoğlu, Çamlı & Çalıkoğlu, 2001).
1-b) Set of RxR	• A plane on which a coordinate system has been determined is called an analytic plane. Each point (P) on the plane is matched with a pair of numbers (a, b), and vice versa. This pair of numbers is written as P (a, b). If $a \in A$ and $b \in B$, where A and B are non-empty sets, then $(a, b) \in AxB$, and a point appears on the plane. If $A = R$ and $B = R$, then one has the following product set: $RxR = R^2 = \{(x, y): x \in R \text{ and } y \in R\}$. Considering that each member of the set of *RxR* is an ordered pair of numbers, each point on the coordinate plane is a member of the set of *RxR*. In other words, the set of RxR is an analytic plane or representing two-dimensional space (Akdeniz, Ünlü & Dönmez, 2012).
2-Rational Numbers	• A number such as$\frac{a}{b}$is called a rational number, where a and b are whole numbers and b $\neq$ 0.The set of rational numbers is denoted by$Q = \left\{\frac{a}{b} l a, b \in Z \text{ } and \text{ } b \neq 0\right\}$. Generally, there is an infinite number of rational numbers between any two rational numbers. However, every point on the number line does not represent a rational number (Komisyon, 2014; Akdeniz, Ünlü & Dönmez, 2012; Akpınar, 2004; Polatoğlu, Çamlı & Çalıkoğlu, 2001). • Since $a = \frac{a}{1}$for $\forall a \in Z$, then $a \in Q$. Therefore,$Z \subset$ Q (Akdeniz, Ünlü & Dönmez, 2012; Balcı, 2012; Akpınar, 2004; Polatoğlu, Çamlı & Çalıkoğlu, 2001). Rational numbers are numbers that can be written as$\frac{p}{q}$,where p and q are whole numbers ($q \neq 0$) since the roots of the equations such as$qx = p$are in the form of $\frac{p}{q}$. If two whole numbers do not have any other common divisor than ± 1, these two numbers are called relatively prime numbers. Since $\frac{p}{q} = \frac{pc}{qc}$is a rational number, where $c \neq 0$is a whole number, p and q are accepted to be relatively prime numbers (Balcı, 2012).
3- Repeating Decimals	• When we divide p or q in the rational number $\frac{\mathrm{p}}{\mathrm{q}}$ ($\mathrm{q} \neq 0$), where p and q are whole numbers, we get the decimal expansion of this rational number, such as $\frac{29}{25} = 1{,}16$, $\frac{1}{3} = 0{,}33$and so forth. As these examples suggest, a rational number can be expressed by finite or infinite decimals. If a portion of an infinite decimal repeats, then it is called a repeating decimal (Akdeniz, Ünlü & Dönmez, 2012). • $\frac{5}{6} = 0{,}8333$ ...As can be seen, the number 3 that follows 8 repeats itself. Such expansions are called repeating decimals, and a short line is drawn above them. An example would be$\frac{5}{6} = 0{,}8333 \ldots = 0{,}8\overline{3}$. Every repeating decimal is a rational number (Polatoğlu, Çamlı & Çalıkoğlu, 2001).

4-Equations	• An equation is a mathematical expression that has an unknownvalue and is confirmed only when specific values are assigned to the unknown value(Tortumlu & Kılıç, 2000) • It is a proposition that involves two mathematical expressions with an equal sign between them. Unknown values are used to find the answer(s) to a given question (Demirtaş, 1986). • It is a mathematical expression that has at least one variable and is confirmed when only certain values are assigned to the variable (Dönmez,2002). • It is an equality with an unknown (Altun, 2010). • 2t+3 and 4t-1 are algebraic expressions where stands for any number and is called a variable. If an algebraic expression is equal to something (e.g. a number or another algebraic expression), it is called an equation. For example, 2t+3 = 4t-1 is an equation (Duffy, Mottershead & Murty, 2003). • If an equality holds between two algebraic expressions, it is called an equation (Baykul, 2009).
5-Inequations	• An expression that can be written as $ax + b > c, ax + b \geq c, ax + b < c, ax + b \leq c$using the symbols $<, \leq, >, \geq$, where $a, b, c \in R \; and \; a \neq 0,$ is called a first-degree inequation with one unknown. A value of x that satisfies an inequation is called the solutions to the inequation, and the set of these solutions is called a solution set. (Kadıoğlu & Kamali, 2013; Polatoğlu, Çamlı & Çalıkoğlu, 2001). • Each of the conditions $P(x) < Q(x)$; $P(x) \leq Q(x) P(x) > Q(x)$; $P(x) \geq Q(x)$, where $P(x)$and $Q(x)$ are two polynomials with real numbers, is called an equation with one unknown. (Akpınar, 2004). • It is an algebraic expression that involves the symbols $<, \leq, >, \geq$and numbers (EBA, 2015).
6-Absolute Value	• The absolute value of $x \in R$is denoted by$\|x\|$. It is defined as $\|x\| = \begin{cases} x, \; if x \geq 0 \\ -x, if x < 0 \end{cases}$. According to this definition, $\|x\| \geq 0$ for each $x \in R$ (Kadıoğlu & Kamali, 2013; Akpınar, 2004). • The absolute value of a number is the distance from the point on the number line that represents that number to the starting point. For example,  Since $\|OB\| = \|-2\| = 2$ and $\|OC\| = \|2\| = 2$, then $\|OB\| = \|OC\|$ (Polatoğlu, Çamlı & Çalıkoğlu, 2001).
7-Relations	• For the non-empty sets A and B, each subset of AxB is called a relation between A and B (or a binary relation) (Akpınar, 2004). • $\beta$is called relation between A and B, where A and B are non-empty sets and $\subset AxB$ (Kadıoğlu & Kamali, 2013).
8-Functions	• A function is a relation that matches each member of a set with a single member of another set. The first set is called the domain whereas the second set is called the range or codomain. The members of the domain are called independent variables, while the members of the range are called dependent variables (Demirtaş, 1986). • The relation f is called a function from A to B if it matches each member of the set A with one and single member of the set B, where A ≠ Ø and B ≠ Ø (MEB, 2012; Akdeniz, Ünlü & Dönmez, 2012; Akpınar, 2004). • A function from A to B a) matches all members of A with members of B b) matches each member of A with a single member of B. If a function matches x with y, it is written as$f: A \rightarrow B, x \rightarrow y = f(x)$, where $x \in A$ and $y \in B$(Akkoç, 2004). • If $f \subset AxB$ meets the following conditions, where A and B are non-empty sets, this relation f is called a function from A to B: a)$(a, b) \in f$F, where there is $b \in B$ for each $a \in A$. b) If$(a, b_1) \in f$and$(a, b_2) \in f$, then $b_1 = b_2$ (Kadıoğlu & Kamali, 2013).

Note:*Natural numbers are denoted by N, whole numbers by Z, rational numbers by Q, irrational numbers by (Q'), real numbers by R and complex numbers by C.*

After the survey was administered to the prospective teachers, each survey form was assigned a number. For example, PT1 denotes prospective teacher 1. Then, their answers to the open-ended questions were scanned and digitalized. The data were analyzed using MAXQDA 11, a qualitative data analysis program. In order to ensure that the data analysis was reliable enough, ten prospective teachers were randomly selected, and their answers were independently assessed by the researcher and a specialist. Using the rubric, the two classified the answers as accurate, incomplete, inaccurate or blank. The similarities and dissimilarities between the answers were identified, and the data were analyzed descriptively (Yin, 1994; Merriam, 1988). The degree of agreement between the coding by the researcher and the specialist was calculated using the following formula: “reliability= (number of agreements) / (number of agreements and disagreements)” (Miles & Huberman, 1994). According to Miles and Huberman (1994), an agreement of 0.70 or higher represents satisfactory reliability. In this study, the inter-rater reliability was 0.83, which is more than satisfactory. The researcher and the specialist met to examine the categories they had generated, clarifying similar categories and discussing dissimilar ones until they reached a consensus (Yin, 1994; Merriam, 1988).

Then, the answers of the remaining prospective teachers were analyzed using the categories generated by the researcher. After the entire dataset was analyzed, these categories, as well as the rubric developed for data analysis, were submitted to two specialists. The categories were revised in accordance with their opinion. The finalized versions are shown here in the form of percentages, frequencies, and direct quotations from the prospective teachers (Tables 3 to 11). The answers were classified as “Accurate-Accurate” if the concept was described and explained accurately, “Accurate-Incomplete” if the concept was described accurately but explained incompletely, “Accurate-Inaccurate” if the concept was described accurately but explained inaccurately and “Accurate-Blank” if the concept was described accurately but no explanation was offered.

3. Findings

The prospective teachers’ conceptions of the mathematical concepts in this study constituted the data, which are presented in Tables 3 to 11.

Table 3 shows the extent to which the participants were able to describe the set of real numbers.

Table 3. Descriptions of the set of real numbers

Concept		**Conceptual Knowledge**		**Procedural Knowledge**					
		f	**%**	**f**	**%**	**Accurate** f-%	**Incomplete** f-%	**Inaccurate** f-%	**Blank** f-%
Set of R (Real numbers)	**Accurate**	2	3.45	29	50	9-15.5	1-1.7	2-3.4	17-29.3
	Incomplete	12	20.7	4	6.9	4-6.9	0	0	0
	Inaccurate	8	13.8	10	17.2	0	0	10-17.2	0
	Blank	36	62.1	15	25.9				
Quotations for Conceptual Knowledge									
<u>Accurate</u> • All numbers between $-\infty$ and $+\infty$(PT11 and 12)									

Incomplete • It is composed of numbers (PT9, 13, 18, 30, 31 and 42). • It is a set that contains rational and natural numbers (PT24). • It is a set that is composed of members such as$\{... - 5 ... 0, . . 2. . \sqrt{2} ...\}$(PT22 and 32). • It is a single-dimensional structure. It is composed of single members (PT7, 41 and 43).
Inaccurate • It is a universal set that encompasses all sets of numbers (PT17, 26, 35and 37). • It is a set that encompasses all sets of numbers except for the set of irrational numbers (PT23). • The set of R contains numbers (PT29). • Real numbers are numbers such as 1, 2 and so forth (PT21).
Quotations for Procedural Knowledge
Accurate-Accurate • The set that encompasses all these numbers is the set of R numbers (PT6, 8, 13, 17 and 26). • Since $-100, -1 \in Z^{-}, 0, 1000 \in N, \frac{3}{5}, 0{,}13\overline{2} \in Q, -\sqrt{2}, \pi \in Q'andN \cup Z \cup Q \cup Q' = R$, these are the members of the set of R (PT7). • Since $N \subset R, Q \subset R, Q' \subset RandZ \subset$, they are members of the set of real numbers (PT32). • They are the members of the set of R since they are rational and irrational numbers (PT9).
Accurate-Incomplete They are real numbers since they are single members (PT41).
Accurate-Inaccurate • Since the set of real numbers encompasses all numbers, they are members of the set of R (PT19). • They are members of the set of real numbers since$-\sqrt{2}$ is irrational and all the other numbers are rational (PT38).
Accurate-Blank • They are members of R (PT1,3, 4, 13-15, 20, 27, 28, 30, 33 -36, 38 and 42).
Incomplete-Accurate • -100, -1, 0 and 1000 are members of the set of whole numbers, $0{,}13\overline{2}$ is a member of the set of decimal numbers and$-\sqrt{2}$ and $\pi$are members of the set of irrational numbers (PT21, 23, 24 and 40).
Inaccurate-Inaccurate • $-100 \in R^{-}$ *and* $-\sqrt{2} \in$ areirrational, $-1 \in R^{-}, 0 \in Z, \frac{3}{5} \in Q$and $0{,}13\overline{2} \in$ arerational, and$\pi \in Q, 1000 \in Z$ (PT5 and24). • They are members of the set of irrational numbers since the set of irrational numbers is the largest set of numbers (PT2, 10, 11, 12and 22-T43). • They are complex numbers because of $\sqrt{2}$ (PT18).
Blank • I can't remember (PT44-58)

The first item of the first question in the first part of the data collection instrument was about the set of real numbers. Only 3.45% of the prospective teachers were able to give accurate answers to this question. More than one-fifth of them (20.7%) gave incomplete answers, and 13.8% gave inaccurate answers. In addition, many of the participants (62.1%) did not answer the question. The corresponding procedural question asked the participants to explain which given numbers were members of which sets. Half of them gave accurate answers. However, inaccurate and incomplete answers were given by 17.2% and 6.9%, respectively (Table 3). The quotations from the participants indicate that they had scientifically inaccurate conceptions and concept images of real numbers.

Table 4 shows the extent to which the participants were able to describe the set of RxR.

Table 4.Descriptions of the set of RxR

Concept		Conceptual Knowledge		Procedural Knowledge					
		f	%	f	%	Accurate f-%	Incomplete f-%	Inaccurate f-%	Blank f-%
The Set of RxR	Accurate	6	10.3	25	43.1	14-24.1	0	1-1.72	10-17.2
	Incomplete	13	22.5	1	1.72	0	0	1-1.72	0
	Inaccurate	12	20.7	6	10.3	0	0	5-8.62	1-1.72
	Blank	27	46.6	26	44.8				

Quotations for Conceptual Knowledge

Accurate
- It is the set of points on the coordinate system (PT17 and 24).
- It is the Cartesian product set R→R (PT29, 39and 42).
- It is the set of points in Euclidean space (PT28).

Incomplete
- It is a set of binary relations (PT18, 21-23, 26, 30, 31, 32, 35, 37, 41 and 43).
- It is the set of non-ordered numbers (PT13).

Inaccurate
- It refers to the numbers between -∞ and +∞ (PT10 and 11).
- It is like the pair of (x, y) (PT4-9 and 14).
- It is composed of the set of R (PT24).
- The result of the set of RxR is a member of the set of R. An example is $-3.4 = -12 \in R$(PT12).
- RxR refers to the real numbers on the coordinate system (PT19).

Blank
- I don't know (PT1-3, 5, 15, 20, 25, 27, 33, 34, 36, 38, 40and 44-T58).

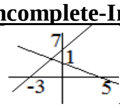
Quotations for Procedural Knowledge

Accurate-Accurate
- They are members of RxR on the rectangular coordinate system (PT17 and 19).
- They are members of the set of RxR since they are composed of ordered pairs (PT14, 22, 24, 26-28, T32-34, 36, 41 and 43).

Accurate-Inaccurate
- These points on the coordinate system are members of the coordinate system, and they are also irrational (PT2).

Accurate-blank
- They are members of R^2(PT3, 4, 13, 15, 29-31 and 40).
- They are members of the Cartesian set RxR (PT35 and 37).

Incomplete-Inaccurate

- 

A(-3, 7) and B(5,1) were incorrectly located on the coordinate system(PT7).

Inaccurate-Inaccurate
- They are complex numbers (PT18).
- A(-3,7) $\in Z$, B(5,1)$\in N$and C($\pi, \sqrt{3}$)∈are irrational (PT23).
- They are real numbers since it encompasses all sets of numbers (PT38 and 39).
- An example is $A(-3,7) \in Q, B(1,5) \in Q$ and $C(\pi, \sqrt{3}) \in Q'$ for $f(1) = a(1) + b$(PT12).

Inaccurate-Blank
- They are real numbers(PT42)

Blank
- I don't know; no idea (PT1, 5, 6, 8, 9, 10, 11, 16, 20, 21, 25 and 44-58).

The second item of the first question in the first part of the data collection instrument was about the set of RxR. Only 10.3% of the prospective teachers were able to give accurate answers. More than one-fifth of them (22.5%) gave incomplete answers, and 20.7% gave inaccurate answers. In addition, many participants (46.6%) did not answer the question. The corresponding procedural question asked the participants to describe what sets included the given points. Less than half of them (43.1%)gave accurate answers. Inaccurate and incomplete answers were given by 10.3%and 1.72%, respectively (Table 4). The quotations from the participants indicate that six of the prospective teachers had scientifically accurate conceptions of the set of RxR, while most of the others had scientifically inaccurate conceptions and concept images.

Table 5 shows the extent to which the participants were able to describe rational numbers.

Table 5.Descriptions of rational numbers

Concept		**Conceptual Knowledge**		**Procedural Knowledge**					
		f	**%**	**f**	**%**	**Accurate**	**Incomplete**	**Inaccurate**	**Blank**
						f-%	**f-%**	**f-%**	**f-%**
Rational Numbers	**Accurate**	4	6.9	6	10.3	2-3.4	0	0	4-6.9
	Incomplete	27	46.6	0	0	0	0	0	0
	Inaccurate	25	43.1	35	60.3	9-16	0	2-3.4	24-41.4
	Blank	2	3.45	17	29.3				

Quotations for Conceptual Knowledge

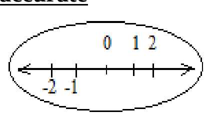
<u>Accurate</u>

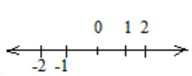
- Numbers that can be written as $\frac{a}{b}$ are called rational numbers, where $a, b \in Z, b \neq 0, (a, b) = 1$ (PT3).
- Each member of the set A that is defined as $A = \left\{\frac{a}{b} / a, b \in Z,\ b \neq 0\ ve(a, b) = 1\right\}$ is called a rational number (PT1 and 4).
- Rational numbers are those numbers that are in the form of $Q = \left\{\frac{a}{b} : a, b \in Z, b \neq 0\right\}$ (PT23).

<u>Inaccurate</u>

- They are numbers that can be written as $\frac{\text{numerator}}{\text{denominator}} = \frac{x}{y}$ (PT5-8, 14, 20-22, 27, 29, 30, 32, 34, 36, 48 and 52).
- They are numbers such as $\frac{5}{2}, \frac{2}{5}, -\frac{3}{5}$and so forth (PT9, 11 and 12).
- The developed form of fractional numbers that have a numerator and a denominator: $\frac{5}{12}, \frac{4}{7}$ and $\frac{12}{10}$ (PT35-37 and 44).
- $Q = \left\{\frac{a}{b} : a, b \in Z, (a, b) = 1\right\}$ (PT31).
- It is a set that encompasses the fractional numbers between $-\infty$ and $+\infty$ (PT58).

<u>Inaccurate</u>

- 

It is a set of rational numbers (PT16).
- They are numbers that can be written as $\frac{a}{b}$, where $a \in Z$ and $b \neq 0 \in R$(PT17 and 19).
- $\frac{a}{b}$ = is a fractional number, where a and b are two whole numbers(PT2).
- Rational numbers are generated when whole numbers are defined as numerators and denominators. Denominators must not be 0. Each number separated by a decimal point is a rational number (PT24).
- Numbers with a numerator and a denominator are rational numbers. Except for 0, all real numbers are also rational numbers (PT33).
- Expressions such as 

$\frac{a}{b}$, where $(b > 0)$ are rational numbers(PT26, 28 and 51).
- All numbers between $-\infty$and$+\infty$(PT40 and 54).
- $R = \{0, 1, 2, \ldots\}$, $R = \left\{1, \frac{1}{2}, \frac{1}{3} \ldots\right\}$ (PT10, 41, 45, 50, 53and 56).
- It is a set whose members are proportional to one another, such as $\frac{1}{2}, 3, -5$and so forth (PT57).
- It is a number generated by proportioning two numbers to one another (PT13,15, 38 and 42).
- Numbers that can be written as$\frac{a}{b}$are called rational numbers, where $a, b \in R$ and $\frac{a}{b} \in R, b \neq 0$,and denoted by Q

(PT25, 43 and 46).

- Qstands for the set of rational numbers. If $\frac{m}{n} \in Z$for$\forall m, n \in Z$, then $m, n \in Q$(PT47 and 49).
- Expressions that can be written as $b > 0, \frac{a}{b}$ (PT26).

Blank

I can't remember (T18 and T55).

Quotations for Procedural Knowledge

Accurate-Accurate

- 0, -7, $\frac{3}{5}$, $0.3\overline{2}$, 0.21, $2\frac{2}{5}$and 3!are rational numbers. $\pi$and$\sqrt{2}$ are not rational since they do not repeat themselves after the decimal point (PT18).
- 0, -7, $\frac{3}{5}$, $0.3\overline{2}$, 0.21, $2\frac{2}{5}$, 3!. $Z \subset Q$since they are members of the set $Q = \left\{\frac{a}{b} I a, b \in Z,\ b \neq 0\right\}$(PT32).

Accurate-Blank

- $\pi$and$-\sqrt{2}$are not rational numbers (PT12).
- 0, -7, $\frac{3}{5}$, $0.3\overline{2}$, 0.21, $2\frac{2}{5}$, and 3! (PT9, 13 and 43)

Inaccurate-Accurate

- 0, -7, $\frac{3}{5}$, $0.3\overline{2}$, π, 0.21, $-\sqrt{2}$, $2\frac{2}{5}$, and 3!. They are all rational and written with a numerator and a denominator (PT36).
- -7, $\frac{3}{5}$, π, 0.21, $-\sqrt{2}$,$2\frac{2}{5}$ and 3! since they can be written as a fractional number (PT3, 7, 22, 29, 37and 41).
- $\frac{3}{5}$, $0.3\overline{2}$, 0.21and $2\frac{2}{5}$ since they have a numerator and a denominator (PT5).

Inaccurate-Inaccurate

- 0, -7, $\underset{+}{\frac{3}{5}}$, $\underset{+}{0.3\overline{2}}$, $\underset{+}{\pi}$,
  $\underset{+}{0.21}$, $-\sqrt{2}$, $\underset{+}{2\frac{2}{5}}$, 3!
  Those numbers with "+" are rational, because they are not whole numbers(PT2).
- 0, -7, $\frac{3}{5}$, $0.3\overline{2}$, π,
  0.21, $-\sqrt{2}$, $2\frac{2}{5}$, 3!
  Except for 0, all of them are rational numbers(PT33).

Inaccurate-Blank

- 0, -7, $\frac{3}{5}$, $0.3\overline{2}$, π,
  0.21, $-\sqrt{2}$, $2\frac{2}{5}$, 3!
  (PT6, 8, 20, 23, 27 and30).
  0, -7, $\frac{3}{5}$, $0.3\overline{2}$, π,
  0.21, $\cancel{-\sqrt{2}}$, $2\frac{2}{5}$, 3!
  (PT10-11, 15, 16, 24, 25, 39and 42).
- 0, -7, $\frac{3}{5}$, $0.3\overline{2}$, π,
  0.21, $-\sqrt{2}$, $2\frac{2}{5}$, 3!
  Irrational
  (PT14, 17, 19, 26, 31, 38 and 40).
  0, $\boxed{-7,}$ $\boxed{\frac{3}{5},}$ $\boxed{0.3\overline{2},}$ π,
  Neutral
  $\boxed{0.21,}$ $\boxed{-\sqrt{2},}$ $\boxed{2\frac{2}{5},}$ $\boxed{3!}$
  (PT28, 34and 35)
- 0, -7, $\frac{3}{5}$, $0.3\overline{2}$, π,
  0.21, $-\sqrt{2}$, $2\frac{2}{5}$, 3! (PT21).

Blank

- I can't remember (PT1, 4and 44-58).

The second question in the first part of the data collection instrument was about rational numbers. Only 6.9% of the prospective teachers were able to give accurate answers. Nearly half of them (46.6%) gave

incomplete responses, and 43.1% gave inaccurate responses. In addition, some participants (3.45%) did not answer the question. The corresponding procedural question asked the participants to explain which of the given numbers were rational. Approximately one-tenth of them (10.3%) gave accurate answers. Many (60.3%) gave inaccurate answers, and 29.3% did not answer the question (Table 5). The quotations from the participants indicate that four of them had scientifically accurate conceptions of rational numbers, whereas most of the others had scientifically inaccurate conceptions and concept images.
Table 6 shows the extent to which the participants were able to describe repeating decimals.

Table 6. Descriptions of the relationship between repeating decimals and rational numbers

Concept		Conceptual Knowledge		Procedural Knowledge					
		f	%	f	%	Accurate	Incomplete	Inaccurate	Blank
						f-%	f-%	f-%	f-%
Repeating Decimals' Relationship to Rational Number	**Accurate**	53	91.4	34	58.6	0	20-34,	0	14-24.1
	Incomplete	0	0	0	0	0	0	0	0
	Inaccurate	0	0	6	10.3	0	6-10.3	0	0
	Blank	5	8.6	18	31				

Quotations for Conceptual Knowledge

Accurate

- Yes, repeating decimals can be written as the proportion of two whole numbers to one another (PT13).
- Yes, they are rational numbers since they can be written as $\frac{a}{b}$ (PT4,7, 26, 27and 57).
- Yes, such as $0.\overline{57} = \frac{57}{900}$. This is a rational number (PT3, 10-12, 17, 21, 28, 33-36, 45, 47, 49, 50, 53and 54).
- A repeating decimal is also a rational number because they can be expressed in the same way (PT22 and 24).
- Yes, we get a rational number if we use the formula or rule for repeating decimals (PT19, 25, 29, 32, 37, 44 and 55).
- Since decimals are rational numbers, repeating decimals are also rational numbers (PT48, 52 and 56).
- Since decimals can be matched with rational numbers, repeating decimals are also rational numbers (PT58).
- Repeating decimals are rational numbers since they are not whole numbers (PT2).
- They are rational numbers since one can add as many zeros as the repeating number repeats itself (PT19).
- They are rational numbers since if the numerator divided by denominator gives you a remainder of a number other than 0, it is a repeating decimal (PT23).
- Yes, repeating decimals are rational numbers even if they can repeat themselves infinitely (PT51).
- They are rational numbers (PT1, 5, 6, 8, 20, 30, 31, 38, 39and 46).

Blank

- I can't remember (PT15, 18 and 40-42).

Quotations for Procedural Knowledge

Accurate-Incomplete

- $0.1\overline{2} = \frac{12-1}{90} = \frac{11}{90}$. Based on the formula for repeating decimals, one can subtract the non-repeating element from the whole number and the result is written as the numerator. Then, one can write as many nines as the repeating number and as many zeros as the non-repeating number, and the result is written as the denominator (PT3, 6-12, 20, 22, 23, 25-27, 29, 32, 33, 38and 41).
- $\frac{12-1}{90} = \frac{11}{90}$. This is how we were taught. This is the formula (PT24).

Accurate-Blank

$\frac{12-1}{90} = \frac{11}{90}$ (PT1, 4, 13-16, 22, 28, 34, 36, 39, 42and 43).

Inaccurate-Incomplete

- $\frac{12-2}{90} = \frac{10}{90} = \frac{1}{9}$. We get the result from the way repeating decimals is written (PT35 and 37).
- $\frac{12}{90}$ (PT7, 19 and 40),
- $\frac{13}{9}$ (PT18)

Blank

- I can't remember(PT2, 30, T31and 44-58).

The third question in the first part of the data collection instrument was about whether repeating decimals are rational numbers, and the participants were expected to explain their answers in detail. Although

91.4% of the prospective teachers were able to give accurate answers, they offered incomplete or inaccurate accounts for their answers. In addition, 8.6% did not answer the question. The corresponding procedural question asked the participants to turn repeating decimals into rational numbers and explain their answers. More than half of them (58.6%)gave accurate answers. However, 10.3% gave inaccurate answers, and 8.6% did not answer the question (Table 6). Although nearly all the participants were aware that repeating decimals are also rational numbers, many of them simply used a memorized formula to turn $0{,}1\overline{2}$ into a rational number and failed to offer any explanations of their answers.

Table 7 shows the extent to which the participants were able to describe absolute value.

Table 7.Descriptions ofabsolute value

Concept		**Conceptual Knowledge**		**Procedural Knowledge**					
		f	**%**	**f**	**%**	**Accurate**	**Incomplete**	**Inaccurate**	**Blank**
						f-%	**f-%**	**f-%**	**f-%**
Absolute Value	**Accurate**	13	22.4	29	50	6-10.3	0	0	23-39.7
	Incomplete	30	51.7	0	0	0	0	0	0
	Inaccurate	10	17.2	9	15,5	0	1-1.72	0	8-13.8
	Blank	5	8.6	20	34.5				

Quotations for Conceptual Knowledge

<u>Accurate</u>
- It is the distance from a number to 0 on the number line (PT16, 17, 19, 38, 39, 41, 42, 48 and 52).
- It is the distance between two expressions (PT14, 15 and 22).
- It is the distance from a number such as $|x| = x$ to 0, where $x \in R$(PT46).

<u>Incomplete</u>
- It is denoted by $|\quad|$. The absolute value of a number is always positive (PT6-8, 10, 47 and 54-58).
- Absolute value makes each member of a range positive (T13).
- It is denoted by $|x|$. Negative numbers are multiplied by (-): $|-4| = -(-4) = +4$ (PT31, 45, 51 and 53).
- It means turning a negative number into positive (PT37).
- They are positive values between certain points (PT44).
- It always denotes positive numbers (PT2, 9, 33 and 50)
- It always denotes a positive number, such as $|x| = x$ and $|-x| = x$ (PT5, 23 and 49).
- It is a concept denoted by $|\quad|$(PT12 and 20).
- It makes symbols $-$and $+$positive under all circumstances(PT24).
- Absolute value means that a number is defined as positive. If a number is between absolute value bars, it is always higher than zero (PT11).
- They are expressions such as $|x - 3|$. A number between the absolute value bars is never negative (PT21).

<u>Inaccurate</u>
- It is a function that turns positive or negative numbers between its bars into positive numbers (PT32).
- It is a place or concept where numbers are always positive (PT34, 35 and 36).
- $-a < |x| < a$, where $a \in R$ and $a \leq 0$(PT1).
- $|a| < x \rightarrow -a < x < a$ (PT3 and4).
- Absolute value is a range (PT25).
- It is the condition that assigns a value in accordance with the symbol between its bars (PT29).
- $|-x| < 0 \ \Rightarrow$is minus x. $|-x| > 0 \ \Rightarrow$ is simply x (PT40).
- If $R \rightarrow R^{+}\{0\}x - y > 0$, then$x - y$, where $x, y \in R$. If $x - y < 0$, then $x + y$, where $x, y \in R$(PT43).

<u>Blank</u>
- I can't remember (PT18, 26, 27, 28 and30).

Quotations for Procedural Knowledge

Accurate-Accurate • Since $x < 3$, then$x - 3 < 0, \|x - 3\| = -(x - 3) = -x + 3$. Since$\big\|- x + 3 + 2x - 2\big\| = \|x + 1\|and\|x + 1\| > 0$, then $\|x + 1\| = x + 1$(PT7, 23, 25, 32, 33 and 41).
Accurate-Blank • $\|-x + 3 + 2x - 2\| = \|x + 1\| = x + 1$ (PT5, 6, 8, 12-22, 26-29, 34-37 and 40).
Inaccurate-Incomplete • If $x = 0$, then $\big\|\|-3\| + (-2)\big\| = 1$ If $x = 1$, then $\big\|\|-2\| + (0)\big\| = 2$ If $x = 2$, then $\big\|\|-1\| + (2)\big\| = 1$(PT1).
Inaccurate-Blank • -x+3-2x=-3x+5 (PT2, 10, 11, 24, 30, 39 and 42).
Blank • I can't remember (PT3, 4, 31, 38 and 43-58).

The fourth question in the first part of the data collection instrument was about absolute value. Only 22.4% of the prospective teachers were able to give accurate answers. More than half of them (51.7%) gave incomplete answers, and 17.2% gave inaccurate answers. In addition, 8.6% did not answer the question. The corresponding procedural question asked the participants to use absolute value and explain their answers. Half of them gave accurate answers. However, 15.5%gave inaccurate answers, and 34.5% did not answer the question (Table 7). The quotations from the participants indicate that nine of them had scientifically accurate conceptions of absolute value, whereas most of the others had scientifically inaccurate conceptions and concept images.

Table 8 shows the extent to which the participants were able describe equations.

Table 8. Descriptions of equations

Concept		Conceptual Knowledge		Procedural Knowledge					
						Accurate	Incomplete	Inaccurate	Blank
		f	%	f	%	f-%	f-%	f-%	f-%
Equation	**Accurate**	11	19	0	0	0	0	0	0
	Incomplete	19	32.8	18	31	2-3.4	3-5.2	0	13-22.4
	Inaccurate	22	37.9	21	36.2	4-6.9	0	4-6.9	13-22.4
	Blank	6	10.3	19	32.8				

Quotations for Conceptual Knowledge
Accurate • They are equalities with one or more unknowns (PT52, 54 and 57). • It is the equality of two quantitative expressions. For example,$a = b$ is an equation (PT1, 17, 19 and 48). • It means equating unknown values with one another (PT42). • They show the equality of two quantitative values, such as $\mathrm{ax + by + c = 0}$, where $\mathrm{a, b} \in \mathrm{R}$(PT3 and 4). • Equation means that two expressions are equal to one another (PT51).
Incomplete • It is a system in which the knowns are used to find out the unknowns (PT2, 24 and37). • An equation needs at least one unknown (PT6,8, 23, 25, 32 and 35). • It is a system with one or more unknowns (PT26, 27, 28, 29, 33, 34, 36, 44and 56). • It is an expression such as ax+b=0, which has constant or variable coefficients (PT40).
Inaccurate • It means equating the right-hand side of an expression with zero and finding out the solution set for the unknown (PT5 and 7). • It is an expression that involves variables such as $x, y$and$z$(PT11). • It is an expression with unknown and constant numbers, such as $x^2 + 5x + 7$ (PT10, 31, 49 and 55).

- It means adding, subtracting, multiplying and dividing unknowns. For example, x+y is an equation (PT46).
- It means that unknown values operate with one another (PT39).
- It means using the known numbers to find out the unknown numbers (PT16, 45 and 53).
- It means equalities or inequalities with unknowns (PT22).
- It means basing a mathematical equality or inequality on a parameter (PT43).

Inaccurate

- It means equating the values on the$x$ and y planein an order. Conditions must satisfy the properties of functions (PT12 and 20).
- It is a method used to achieve a goal (PT14 and15).
- An equation needs at least two unknowns. These unknowns must be related to one another (PT47).
- They are numbers with two or more unknowns (PT50).
- It means expressing two or more concepts with an operation (PT58).

Blank

- I can't remember (PT13, 18, 21, 30, 38 and 41).

Quotations for Procedural Knowledge

Incomplete-Accurate

- **b)** $y = 2x - 3$
  c) $3x - 5 = 0$
  d) $x = 0$ since there is an unknown and equality (PT35 and 32).

Incomplete-Incomplete

- **b)** $y = 2x - 3$
  c) $3x - 5 = 0$
  d) $x = 0$
  e) $(x-2)^2 = x^2 - 4x + 4$since an unknown is equated with a number(PT1 and 23).
- **c)** $3x - 5 = 0$
  d) $x = 0$are equations since there is a solution to them. The others are functions. They are inequations (PT18).

Incomplete-Blank

- **b)** $y = 2x - 3$
  c) $3x - 5 = 0$
  e) $(x-2)^2 = x^2 - 4x + 4$ (PT3, 16, 33, 38, 40 and 43).
- **b)** $y = 2x - 3$
  c) $3x - 5 = 0$ (PT20, 21, 27, 30, 31, 39 and 42).

Inaccurate-Accurate

- **a)** $2x^2 - x + 2$. This is not an equation since it is not equal to anything (PT 6 and 8).
- **a)** $2x^2 - x + 2$
  b) $y = 2x - 3$
  c) $3x - 5 = 0$
  f) $2x - 2y < 4$. These are equations since there is at least one unknown (PT25 and 29).

Inaccurate-Inaccurate

- **b)**$y = 2x - 3$
  f)$2x - 2y < 4$since they have two unknowns (PT2).
- **a)** $2x^2 - x + 2$
  e)$(x-2)^2 = x^2 - 4x + 4$. They are quadratic equations with one unknown (PT10).
- **d)**Except for$x = 0$, all the others are equations. This is simply a point according to the definition of equations. It is not a function (PT24).
- **a)** $2x^2 - x + 2$ is a quadratic equation with one unknown since it is second-degree (PT36).

Inaccurate-Blank

- **b)**$y = 2x - 3$
  c) $3x - 5 = 0$
  d) $x = 0$
  f) $2x - 2y < 4$ (PT5 and 22).
- **a)** $2x^2 - x + 2$
  b) $y = 2x - 3$

c) $3x - 5 = 0$ **e)** $(x-2)^2 = x^2 - 4x + 4$ (PT9, 11, 12, 14, 15, 17, 19,34, 37 and 41). • **b)** $y = 2x - 3$is not an equation(PT26).
Blank • I can't remember (PT13, 14and 44-58).

The fifth question in the first part of the data collection instrument was about equations. The participants were also asked to define an equation. Only 19% of the prospective teachers were able to give accurate answers. Nearly one-third of them (32.82%) gave incomplete responses, and 37.9% gave inaccurate responses. In addition, 10.3% did not answer the question. The corresponding procedural question asked the participants to select equations from the given alternatives and explain their answers. None of them gave an accurate answer. However, 31%gaveincompleteanswers, and 36.2% gave inaccurate answers. In addition, 32.8% did not answer the question(Table 8). The quotations from the participants indicate that three of them had scientifically accurate conceptions of equations, whereas most of the others had scientifically inaccurate conceptions and concept images.

Table 9 shows the extent to which the participants were able to describe inequations.

Table 9.Descriptions of inequations

Concept		**Conceptual Knowledge**		**Procedural Knowledge**					
		f	**%**	**f**	**%**	**Accurate f-%**	**Incomplete f-%**	**Inaccurate f-%**	**Blank f-%**
Inequation	**Accurate**	15	25.9	8	13.8	0	0	0	8-13.8
	Incomplete	2	3.45	0	0	0	0	0	0
	Inaccurate	13	22.4	33	56.9	1-1.72	0	5-8,62	28-48.3
	Blank	28	48.3	16	27.6				
Quotations for Conceptual Knowledge									
Accurate • It is a condition in which one of two given unequal values can be labelled as bigger or smaller (PT12 and 32). • It refers to numerical expressions that contain the symbols$>$, $\geq$, $<$ and $\leq$. Inequality persists when the same number is added to or subtracted from both sides or when they are multiplied or divided by a positive number (PT17 and 19). • It refers to expressions such as $ax + b \leq c$ (PT21 and 22). • They are expressions such as $a \leq x \leq b$, which determines a range for the value of the unknown (PT33, 34, 35 and 36). • A relation such as $x > y$ is an inequation where $x, y \in R$ and $\|x - y\| \neq 0$ (PT43). • $3x + 5y \geq 7$ $0 \leq 6z + 11y \leq 14$ (PT1, 3, 4 and 11).									
Incomplete • They are unequal expressions (PT10). • We can say that inequations are within a certain range. For example, if $2 < x < 5$, then x is a number between 2 and 5 (PT5).									

Inaccurate

- Lack of equality. In other words, they are expressions with no equality. Such as$<, >$(PT26 and 42)
- It means finding out a solution set in accordance with the range of a number using symbols such as >and< (PT7).
- It means a value assigned within a range (PT6, 8and 29).
- It refers to terms by which a certain value can be ordered equally (PT9).
- It means not knowing the actual value of an expression and assigning an approximate value (PT14 and 15).
- When ranges are used for determining the root of an equation, it is an inequation (PT23and 25).
- Each equation is an inequation (PT24).
- It is an expression that determines the range of the value for an unknown in a system such as $ax + b = 0$ (PT37).

Blank

- I don't know (PT2, 13, 16, 18, 27, 28, 30, 31, 38, 39, 41 and 44-58).

Quotations for Procedural Knowledge

Accurate-Blank

- $\mp 2 < x < \mp 5$, $x \in (-5, -2) \cup (2, 5)$ (PT16, 18, 30, 31, 32, 38, 42 and 43)

Inaccurate-Accurate

- Based on the definition of absolute value, x^2 is treated as the absolute value. $|2| < x < |5|$, $-2 < x < 2 - 5 < x < 5$. Therefore, the result is $-2 < x < 5$(PT24).

Inaccurate-Inaccurate

- Since $-2^2 < x^2 < 5^2$, the largest domain is between -2 and 5 (PT26).
- When we take the square root of both sides, the result is $2 < x < 5$ (PT14, 15and 17).

Inaccurate-Blank

- $4 < x^2$, $x^2 < 25, x > 2$, or $x > -2, x < 5$ or $x < -5$ (PT27 and 28)
- If $\sqrt{4} < \sqrt{x^2} < \sqrt{5}$, then $2 < x < 5$ and x=3, 4 (PT1, 2, 3, 4, 5, 6, 7, 8, 19 and 20).
- $\mp 2 < x^2 < \mp 5$, $(-5, 5)$ (PT9, 11, 12, 25, 29, 33, 34, 35, 36, 37 and 40).
- $-2 < x < 5$(PT10 and 13).
- $(2, 5)$, real numbers between 2 and 5 (PT22 and 39).
- If $\sqrt{4} < \sqrt{x^2} < \sqrt{5}$, then $2 < x < 5$, (PT23 and 41).

Blank

- I can't remember (PT21 and 44-58).

The sixth question in the first part of the data collection instrument was about inequations. The participants were also asked to define an inequation. Only 25.9% of the prospective teachers were able to give accurate answers. Some (3.45%) gave incomplete answers, and 22.4% gave inaccurate answers. In addition, 43.8% did not answer the question. The corresponding procedural question asked the participants to determine the largest domain of a given inequation and explain their answers. Only 13.8% of them gave accurate answers. However, 56.9% gave inaccurate answers. In addition, 27.6% did not answer the question (Table 9).

Table 10 shows the extent to which the participants were able to describe relations.

Table 10. Descriptions of relations

Concept		**Conceptual Knowledge**		**Procedural Knowledge**					
		f	**%**	**f**	**%**	**Accurate**	**Incomplete**	**Inaccurate**	**Blank**
						f-%	**f-%**	**f-%**	**f-%**
Relation	**Accurate**	12	20.7	24	41.4	0	2-3.45	2-3.45	19-32.8
	Incomplete	10	17.2	13	22.4	0	0	5-8.62	8-13.8
	Inaccurate	26	44.8	3	5.17	0	0	0	3-5.17
	Blank	10	17.2	18	31				
Quotations for Conceptual Knowledge									

Accurate

- Each subset of the Cartesian product set BxA is called a relation between B and A, where A and B are non-empty sets (PT45).
- Each subset of the set AxB is a relation (T1, T3, T4, T13, T19, T48 and T50).
- Different subsets of the Cartesian product composed of ordered pairs are called a relation (PT27, 28, 29 and 30).

Incomplete

- It is a set of ordered pairs (PT9, 14, 15, 23, 43 and 57).
- It is composed of pairs defined as $A \rightarrow B$(PT18, 31 and 51).
- It is an expression that results from the Cartesian product (PT37).

Inaccurate

- It is a special state of a function. It is composed of ordered pairs (PT17).
- A function in the form of $\beta: AxB \rightarrow AxA,\ \beta = \{(x, y): x \in A \text{ and } y \in B\}$is called a relation(PT22).
- It is a relation function (PT5, 6 and 8).
- A function that has reflexive, symmetric and transitive properties is called a relation (PT10, 11 and 12).
- It is a relationship between two sets (PT40).
- It is a relationship between two expressions. In order for a relation to exist, there must be a function between the two expressions (PT44).
- It is composed of ordered pairs, such as $\beta = \{\forall x \in \beta, \forall y \in \beta \text{ için } x, y \in \beta\}$. These ordered pairs must be related to one another (PT47).
- The continuous function of the Cartesian product is called a relation (PT53).
- A rule defined on a given set is called a relation (PT46).
- Each set that goes from the set A to the subset B is called a relation from A to B (PT55).
- It is a set that contains a combination of the Cartesian products (P56).
- It is the Cartesian product of expressions that involve ordered pairs (PT25 and 26).
- A R defined between $(a, b)$and$(c, d)$is called a relation (PT32).
- The Cartesian product of two functions is called a relation (PT33, 34, 35and 36).
- It means systematic operations between two sets (PT42).
- The expression mxnis called a relation, where $\{m, n \in N\}$ (PT49).
- It shows that at least two expressions are related to one another. At least two expressions are required (PT54).
- If there is at least one $b \in B$for $a \in A$, then this is called a relation (PT58).

Blank

- I can't remember (PT2, 7, 16, 20, 21, 24, 38, 39, 41and 52).

Quotations for Procedural Knowledge

Accurate-Incomplete

- **b)** $\beta_2 = \{(-3, 1),\ (-3, 2), (-3, 3)\}$
  c) $\beta_3 = \{(-3, 1)\}$

d) $\beta_4 = \{(-3, 1),\ (0, 3)\}$. These are relations since a member of the set A and a member of the set B form a binary relation (PT18 and 25).

Accurate-Inaccurate

- **b)** $\beta_2 = \{(-3, 1),\ (-3, 2), (-3, 3)\}$
  c) $\beta_3 = \{(-3, 1)\}$
  d) $\beta_4 = \{(-3, 1),\ (0, 3)\}$since they go to one member (PT9 and 17).

Accurate-Blank

- **b)** $\beta_2 = \{(-3, 1),\ (-3, 2), (-3, 3)\}$
  c) $\beta_3 = \{(-3, 1)\}$
  d) $\beta_4 = \{(-3, 1),\ (0, 3)\}$(PT5, 6, 7, 12, 13, 19, 21, 22, 23, 24, 25, 26, 30, 31, 32, 35, 36, 38, 39 and 43)

Incomplete-Inaccurate

- **b)** $\beta_2 = \{(-3, 1),\ (-3, 2), (-3, 3)\}$since all members of the set B must be used (PT3).

• **b)** $\beta_2 = \{(-3,1),\ (-3,2),(-3,3)\}$since it has reflexive, symmetric and transitive properties (PT10, 11, 14and 37).
Incomplete-Blank **b)** $\beta_2 = \{(-3,1),\ (-3,2),(-3,3)\}$ (PT1, 4, 8, 15, 16,27, 28 and 29)
Inaccurate-Blank • **a)** $\beta_1 = \{(-3,1),\ (2,-2)\}$ **b)** $\beta_2 = \{(-3,1),\ (-3,2),(-3,3)\}$ **c)** $\beta_3 = \{(-3,1)\}$ **d)** $\beta_4 = \{(-3,1),\ (0,3)\}$ **e)** $\beta_5 = \{-3,-2,0,\ 1,\ 2,\ 3\}$ (PT2, 20 and 34).
Blank • I can't remember (PT33, 40, 41 and 44-58).

The seventh question in the first part of the data collection instrument was about relations. The participants were also asked to define a relation. More than one-fifth of the prospective teachers (20.7%) were able to give accurate answers. Less than one-fifth of them (17.2%) gave incomplete answers, and 44.84% gave inaccurate answers. In addition, 17.2% did not answer the question. The corresponding procedural question asked the participants to select relations from the given alternatives and explain their answers. Less than half of them (41.4%) gave accurate answers. However, 22.4% gave incomplete answers, and 5.17% gave inaccurate answers. In addition, 31% did not answer the question (Table 10).

Table 11 shows the extent to which the participants were able to describe functions.

Table 11.Descriptions of functions

Concept		Conceptual Knowledge		Procedural Knowledge					
		f	%	f	%	Accurate f-%	Incomplete f-%	Inaccurate f-%	Blank f-%
Function	Accurate	10	17.2	23	39.7	1-1.72	0	0	22-37.9
	Incomplete	15	25.9	6	10.3	1-1.72	0	1-1.72	4-6.9
	Inaccurate	30	51.7	10	17.2	0	4-6.9	1-1.72	5-8.62
	Blank	3	5.17	19	32.8				

Quotations for Conceptual Knowledge
Accurate • It is a special case of a relation. No member of the domain can be unmatched. Instead, each member must be matched with a member of the range (PT2, 26, 29and 30). • The relation f that matches each member of A to a single member of B is called a function from A to B, where A and B are non-empty sets (PT48). • No member of the domain can be unmatched. A member of the domain can be matched with only one member of the range (PT17, 19, 21 and 23). • The expression $f: R \to R^2 x, y \to f(x,y) = x^2 + y^2$is called a$R \to R^2$function (PT49).
Incomplete • It is a special case of a relation. In order for it to be called a function, there must be a domain and a range (PT3, 4, 13 and 14). • A function is a relation. It is a transformation. There must be a relation between two sets (PT24). • It means one single match between two sets. For example, it is called a function if each child in the domain has a mother (PT44 and 46). • If $f: A \to B$is a function, no member of the domain A can be unmatched (PT47). • Members must look different in the domain and in the range (PT33). • A function is a special case of a relation. There must be two sets, namely one domain and one range, and no member of the former will be unmatched (PT34, 35and 36).

- A function is a special case of a relation (PT37).
- It is a special case of a relation. Each member must be matched with one single member (PT43).
- We get an a from the set A and a b from the set B, and then it is a A → B relation based on a certain rule (PT1).

Inaccurate

- If no member of A is unmatched, then it is called a function (T58).
- It means that a member of the domain is matched with a member of the range (PT7, 15, 25, 27, 28, 50and 57).
- It is an equation where all members of the domain are matched with the members of the range (PT18).
- It is a relation with ordered pairs (T9).
- $f: A \rightarrow B$is a relation where no member of the domain will be unmatched(PT31 and 32).
- It means making new products out of previous products in a factory (PT41).
- A relation from A to B is called a function, where A and B are sets (PT42).
- It is a transformation from one set to another (PT51 and 53).
- If each member of the set A corresponds to a single member of the set B and no member of the latter is unmatched, it is a function (PT55).
- Function is matching members of the domain with members of the range so that there will be no unmatched members in the latter (PT56).
- Each member of the range must be matched with one member of the domain, and it must be a covering function (PT45).

Inaccurate

- It is an expression such as $f(x) = mx + n$ (T5, T6, T8 and T40).
- It is an expression that satisfies the following: $f: A \rightarrow B\ f(x) = y$ (PT10, 11, 12, 16, 20, 22and 54).

Blank

- I can't remember (PT38, 39and 52)

Quotations for Procedural Knowledge

Accurate-Accurate

- **c)** $\{(0,1),(2,1),\ (4,\ 1)\}$
  d) $\{(0,2),(2,1),(4,\ 1)\}$
  e) $\{(0,1),(2,\ \ 3),\ (4,\ 2)\}$ These are functions since no member of the domain is unmatched and each member of the domain is matched with one single member of the range (PT16).

Accurate-Blank

- **c)** $\{(0,1),(2,1),\ (4,\ 1)\}$
  d) $\{(0,2),(2,1),(4,\ 1)\}$
  e) $\{(0,1),(2,\ \ 3),\ (4,\ 2)\}$(PT2, 9, 10, 12, 13, 14, 17, 19, 21, 23, 24, 25, 28, 30, 31, 32, 33, 34, 35, 36, 37and 43).

Incomplete-Accurate

e) $\{(0,1),(2,\ \ 3),\ (4,\ 2)\}$. It is e.No member of the domain can be unmatched(PT4)

Incomplete-Inaccurate

- **e)** $\{(0,1),(2,\ \ 3),\ (4,\ 2)\}$. It is e.No member of the range can be unmatched (PT3).

Incomplete-Blank

- **e)** $\{(0,1),(2,\ \ 3),\ (4,\ 2)\}$. It is e.(PT15, 22, 27 and 18).

Inaccurate-Incomplete

- **b)** $\{(\mathbf{1},\mathbf{0}\ ),(\mathbf{2},\mathbf{4}),(\mathbf{2},\mathbf{2}))\}$. Not b since it goes from B to A(PT6).
- **a)** $\{(0,1),(0,2),(2,1),\ (4,\ 1)\}$
  e) $\{(0,1),(2,\ \ 3),\ (4,\ 2)\}$. It is a and e since they go from A to B and no member of A is unmatched (PT7).
- **a)** $\{(0,1),(0,2),(2,1),\ (4,\ 1)\}$
  c) $\{(0,1),(2,1),\ (4,\ 1)\}$
  d) $\{(0,2),(2,1),(4,\ 1)\}$
  e) $\{(0,1),(2,\ \ 3),\ (4,\ 2)\}$since each member of the domain is matched with the same or different member of the range (PT8).
- **a)** $\{(0,1),(0,2),(2,1),\ (4,\ 1)\}$
  c) $\{(0,1),(2,1),\ (4,\ 1)\}$
  d) $\{(0,2),(2,1),(4,\ 1)\}$
  e) $\{(0,1),(2,\ \ 3),\ (4,\ 2)\}$since b goes from B to A. In a, c, d and e, it goes from the set A to the set B. (PT20).

Inaccurate-Inaccurate • **a)** $\{(0,1),(0,2),(2,1),\ (4,\ 1)\}$ **c)** $\{(0,1),(2,1),\ (4,\ 1)\}$ **d)** $\{(0,2),(2,1),(4,\ 1)\}$ **e)** $\{(0,1),(2,\ \ 3),\ (4,\ 2)\}$since they are composed of the sets A and B(PT39).	**Inaccurate-Blank** • **a)** $\{(0,1),(0,2),(2,1),\ (4,\ 1)\}$ **c)** $\{(0,1),(2,1),\ (4,\ 1)\}$ **e)** $\{(0,1),(2,\ \ 3),\ (4,\ 2)\}$ (PT5,11, 29and 42). • **a)** $\{(0,1),(0,2),(2,1),\ (4,\ 1)\}$ **c)** $\{(0,1),(2,1),\ (4,\ 1)\}$ (PT26).
Blank • I can't remember (PT1, 38, 40, 41and 44-58).	

The final question in the first part of the data collection instrument was about the description of functions. The participants were also asked to describe a function. Less than one-fifth of the prospective teachers (17.2%) were able to give accurate answers. More than a quarter of them (25.9%) gave incomplete answers, and 51.7% gave inaccurate answers. In addition, 5.17% did not answer the question. The corresponding procedural question asked the participants to select functions from the given alternatives and explain their answers. More than one-third of them (39.7%) gave accurate answers. However, 10.3% gave incomplete answers, and 17.2% gave inaccurate answers. In addition, 32.8% did not answer the question (Table 11). The quotations from the participants indicate that five of them had scientifically accurate conceptions of functions, whereas most of the others had scientifically inaccurate conceptions and concept images.

5. Discussion, Conclusion and Implications

A large majority of the prospective teachers gave either incomplete or inaccurate answers to the conceptual question about the set of real numbers or skipped the question and offered no descriptions. This suggests that they had difficulty describing the set of real numbers. Many prospective teachers described the set of real numbers either too broadly or too narrowly. Although many participants gave accurate answers to the procedural question about the set of real numbers, more than one-third either did not explain their answers or offered incomplete or inaccurate explanations. In addition, nearly a quarter of the participants gave inaccurate answers and offered inaccurate explanations for them.

The quotations that were grouped under the categories of inaccurate answers (Table 3) indicate that the prospective teachers had inaccurate conceptions and concept images of real numbers. Examples include“$Q \cup N$, " "$Z \cup Q'$, "“C;”and “$C - Q'$, "” set of numbers” and “Z^+.” Only two prospective teachers had scientifically accurate conceptions and concept images of real numbers since they described the concept as “all numbers between $-\infty$and $+\infty$.”

Similarly, a large majority of the prospective teachers described the set of RxR either incompletely or inaccurately or skipped the question and offered no descriptions. This suggests that they had difficulty describing the set of RxR. A significant number of them wrongly thought that the set of RxR is the same as R (the set of real numbers). Although nearly half of the participants gave accurate answers to the procedural question, many of them did not explain their answers or offered inaccurate explanations. In addition, a significant number of participants gave inaccurate answers and offered inaccurate explanations, and many prospective teachers did not answer the question. All these findings indicate that the prospective teachers were not competent to describe the set of real numbers and RxR and failed to offer a formal definition.

Only a few participants had scientifically accurate conceptions and concept images of the set of RxR (Table 4). Their answers included: “the set of points on the coordinate system” and “the Cartesian product set R→ R.” On the other hand, many prospective teachers had scientifically inaccurate conceptions and concept images of the set of RxR. Examples of their answers include “the set of binary relations,” “the set of non-ordered numbers,” “numbers between -∞ and +∞,” “the pair of (x, y)” and “RxR=R.” The reason for these inaccurate conceptions and concept images may be epistemological obstacles to teaching the concept (Soylu, Akgün, Dündar & İşleyen, 2011). Another reason is that teachers tend to focus on technical information and explanations when they teach concepts such as the set of natural numbers, the set of rational numbers, the set of irrational numbers, the set of real numbers and the set of complex numbers. In other words, students' internalization of number systems is hindered by the fact that information is simply presented to be memorized and they are not encouraged to think (Çiftçi, Akgün & Soylu, 2015). Some of the prospective teachers offered a narrow description of the set of real numbers, considering it to be a combination of rational and natural numbers or to be the set of natural numbers. Others described the concept too broadly since they considered it to be the same as the set of complex numbers. Apparently, these students were unable to internalize these concepts from their elementary education to university education. This finding is supported by the results of a study of high school students and prospective teachers by Fischbein, Jehiam and Cohen (1995). They reported that not all the participants were able to describe rational numbers, irrational numbers and real numbers accurately and that many failed to distinguish among different examples of whole numbers, rational numbers, irrational numbers and real numbers.

Nearly all the prospective teachers described the set of rational numbers either incompletely or inaccurately or skipped the question and offered no descriptions. This suggests that they had difficulty describing the set of rational numbers. Examples of these answers include: "factional numbers with a numerator and a denominator," "a set that encompasses the fractional numbers between $-\infty$and $+\infty$," "numbers such as $\frac{a}{b}$, where $a \in Z$ ve $b \neq 0 \in R$," "$\frac{a}{b} =$ non-whole numbers, where a and b are two whole numbers," "all numbers between $-\infty$ and$+\infty$, "a number generated by proportioning two numbers to one another, "numbers such as $\frac{a}{b}$, where $a, b \in R$ and $\frac{a}{b} \in R, b \neq 0$" and "numbers that can be written as $\frac{a}{b}$, where $b > 0$." Nearly half of the prospective teachers described the set of rational numbers inaccurately. Some confused the set of rational numbers with the set of whole numbers, whereas others wrongly considered the set of rational numbers to be the same as real numbers or did not fully know about the necessary conditions for a rational number. In addition, only a few participants were able to select rational numbers from the given alternatives. However, they were unable to explain their answers. Nearly one-third of them did not answer this procedural question. All these things suggest that they had difficulty describing rational numbers.

Nevertheless, nearly all the participants knew that repeating decimals are also rational numbers. Even so, more than half of them explained their answers incompletely, and 10% offered inaccurate explanations. In addition, nearly 5% did not explain their answers. Apparently, they did not know about the conceptual relationship between repeating decimals and rational numbers. Similarly, although nearly two-thirds of the participants gave accurate answers to the procedural question about repeating decimals, one-third of them explained their answers incompletely and around a quarter of them were unable to offer any explanations. This also suggests that they had difficulty explaining the conceptual relationship between repeating decimals and rational numbers. In addition, one-third of the participants skipped this procedural question. A look at the quotations from the participants who gave accurate answers both to the conceptual and procedural questions, but explained their answers incompletely shows that they had simply memorized the rule used for translating repeating decimals into rational numbers.

Students start learning about rational numbers and their relationships with repeating decimals in secondary school (Durmuş, 2005). Then, they frequently encounter the concept in their high school and university education and use it for learning other mathematical topics and solving problems. Therefore, prospective mathematics teachers who are about to graduate from university are naturally expected to offer a formal definition of the concept rather than their personal conceptions. However, the participants were not competent to describe rational numbers and failed to offer a formal definition of it. In other words, they failed to express the necessary conditions for rational numbers. The failure of the participants might have resulted from their previous teachers' emphasis on operations rather than the wide variety of meanings of the concept of rational number (Moss & Case, 1999). Students need to understand different problems when they try to use natural numbers to deal with rational numbers. Teachers tend to ignore the basic properties of rational numbers by simply focusing on the fact that it is a fractional number and using procedural knowledge for solving problems (Durmuş, 2005). Many studies in the literature (İpek, Işık & Albayrak, 2005; Toluk, 2002; Aksu, 1997; Mack, 1995; Simon,1993; Behr, Harel, Post & Lesh, 1992) have reported that students have problems with rational numbers because of its many meanings and complexity. One of the main reasons for their problems might be that they are simply presented with rules, formulas and relationships and encouraged to learn them through memorization. This is despite the fact that procedural and conceptual knowledge should be balanced in classes and discussions should be organized to enable them to construct meanings and relationships with other concepts.

Although a quarter of the prospective teachers were able to describe absolute value accurately, incomplete and inaccurate descriptions were offered by more than half of them and nearly one-fifth of them, respectively. Half of them gave accurate answers to the procedural question about absolute value, but a significant number of them were not able to explain their answers. Some participants said that absolute value is denoted by $| \quad |$and is always positive. An example of the answers by the participants who had scientifically inaccurate conceptions and concept images of absolute value is "it is denoted by $| \quad |$and the number between absolute value bars is never negative." In addition, one participant considered absolute value to be "a function that turns positive or negative numbers into positive." The quotations in the category Inaccurate-Inaccurate suggest that the prospective teachers had difficulty using algebraic expressions and described absolute value inaccurately.

A considerable number of participants gave inaccurate answers to the procedural question about absolute value. In addition, more than one-third skipped the question. As can be concluded from Table 7, some prospective teachers gave incomplete answers to the conceptual question but offered accurate explanations. This indicates that they had memorized the properties of absolute value but had difficulty describing the concept. This finding is supported by the findings in the literature. For example, Basturk (2004) conducted a study with high school students and observed that they had a variety of mistakes and simply ignored the absolute value when they were solving problems that were associated with absolute value. Similarly, Yenilmez and Avcu (2009) carried out a study with elementary and secondary school students. They reported that the students had difficulty with the absolute value of lettered expressions and in equations that involved the absolute value. Ciltas, Isik and Kar (2010) worked with prospective elementary school mathematics teachers. The authors reported that the participants failed to make geometric interpretation of the absolute value, that memorized knowledge from high school was prevalent in the answers to the procedural test and that they did not fully comprehend the definition of absolute value. In the Turkish educational system, students encounter absolute value, though at the level of definition, for the first time in the sixth grade level. They intensively use the concept for arithmetic operations. In addition, they use the concept arithmetically or algebraically for learning other mathematical concepts in high school and at university. One reason why students have difficulty with the concept despite their frequent encounter with it during their high school or university education may be that absolute value is abstract by nature. The difficulties and problems can be avoided if abstract mathematical concepts are concretized and taught using concrete tools (Baykul, 1999).

Although nearly a quarter of the prospective teachers described the concept of equation accurately, more than one-third of them explained their answers incompletely or inaccurately. In addition, nearly half of the participants described the concept inaccurately. Some participants considered an equation to be "the equality of two expressions", ignoring the required properties of these expressions. Others had scientifically inaccurate conceptions and concept images. Examples of these answers by include: "expressions such as$ax + by + c = 0$, where $a, b \in R$, "a system for finding out the unknown by using the knowns," "algebraic expressions with variables or unknowns," "finding out the unknown," "mathematical equalities or inequalities," "a method for achieving a goal" and "equating values on the x and y plane in an order; conditions must satisfy the properties of a function."

More than one-third of the prospective teachers gave incomplete answers to the procedural question that required them to select equations from the given alternatives. In addition, more than one-third gave inaccurate answers, and more than one-third skipped the question. Those who gave incomplete or inaccurate answers to the procedural question did not explain their answers, which suggest that they had difficulty describing equations. A significant number of prospective teachers knew about the properties of equations but had difficulty explaining them (Table 8). Similarly, Lima and Tall (2006) found that students have difficulty understanding the concept of an equation, although it is one of the most significant mathematical concepts and one of the most important components of mathematical thinking. They attributed the students' difficulty in mathematical calculations to their weak understanding of the concept of an equation, which, they thought, could weaken their problem-solving skills. Likewise, Dane and Baskurt (2012) carried out a study with eight graders and found that they were not able to describe the concept of an equation accurately. Similar findings were reported by Aydin and Kogce (2008), who conducted a study with prospective elementary, secondary and high school teachers.

Nearly a quarter of the prospective teachers described the concept of inequation accurately. However, almost one-fifth of them described the concept inaccurately, and nearly half of them did not offer any descriptions. Nearly half of the prospective teachers reported that they had no idea what an equation was and did not offer any explanations. According to the quotations, some wrongly thought that inequations must be first degree with one unknown. Examples of conceptions and concept images by other participants include "a range," "an equation," "values assigned within a range," and "not knowing the actual value of an expression and assigning an approximate value." Similarly, more than half of the participants gave inaccurate answers to the procedural question about inequations and offered no explanations. In addition, nearly one-third did not answer the question. As can be concluded from the answers to the conceptual question (Table 9), the prospective teachers knew about the properties of inequations, but had difficulty explaining them.

Although nearly a quarter of the prospective teachers described the concept of relation accurately, nearly a quarter of them described it incompletely, and nearly half of them described it inaccurately. Nearly all the participants who described the concept accurately offered incomplete explanations. In addition, nearly one-fifth of them did not offer any explanations. In other words, nearly four-fifths of them were not able to describe the concept of relation properly. Only one participant was able to offer a description that complied with the scientific definition of the concept. The others had either incomplete or inaccurate conceptions of it. Some had scientifically inaccurate conceptions and concept images since they described the term as "the set of ordered pairs," "pairs defined as$A \rightarrow B$," or "an expression that results from the Cartesian product," thereby neglecting the properties that an expression must have to be called a relation. Examples of other scientifically inaccurate conceptions or concept images include "a function with certain properties," "relationship between two sets," "each set that goes from any set of A to the subset of B," "a set that contains a combination of the Cartesian products," "the Cartesian product of expressions that involve ordered pairs," "a R defined between (a, b) and (c, d)," "mxn,where $m, n \in N$," "at least two expressions are related to one another and at least two expressions are required" and "there must be at least one $b \in B$ for $a \in A$."

When the participants were asked to select relations from the given alternatives and explain their answers, nearly two-fifths of them gave accurate answers. However, more than two-thirds of them did not offer any explanations, and the others explained their answers either incompletely or inaccurately. In addition, nearly a quarter of them gave incomplete answers to the procedural question about relations, and a significant number of them did not offer any explanations. Also, one-third of them skipped the question. All these thing suggest that the participants had difficulty describing relations.

Although nearly one-fifth of the prospective teachers described the concept of function accurately, nearly a quarter of them described the concept incompletely, and more than half of them described it inaccurately. Half of those who described it accurately explained their answers accurately, whereas the other half was unable to do so. In other words, nearly four-fifths of the participants either gave incomplete or inaccurate answers to the question about the description of functions or skipped the question. The quotations from the participants show that only five of them were able to offer descriptions that complied with the scientific definition of the concept. The others had either incomplete or inaccurate conceptions of it. Some were unable to explain the required properties of a function fully. Their answers suggest that they had scientifically inaccurate conceptions or concept images of functions: “a special case of relation,” “there must be a domain and a range,” “there must be a relation between two sets, “one single match between two sets,” “if $f: A \rightarrow B$ is a function, no member of the domain A can be unmatched,” “members must look different in the domain and in the range” and “each member must be matched with one single member.”

Some prospect.ve teachers neglected the properties that an expression must have to be called a function, and their answers suggest that they had scientifically inaccurate conceptions or concept images: “If no member of A is unmatched, then it is called a function,” “member of the domain is matched with a member of the range,” “an equation where all members of the domain are matched with members of the range,” “a relation with ordered pairs,” “making new products out of previous products in a factory,” “a relation from the set A to the set B, “a transformation from one set to another,” “if each member of the set A goes to one single member of the set B and no member of the latter is unmatched, it is a function,” “matching members of the domain with members of the range in a way there will be no unmatched members in the latter,” “each member of the range must be matched with one member of the domain and it must be a covering function,” “expressions such as $f(x) = mx + n$” and “expressions that satisfy the criterion $f: A \rightarrow Bf(x) = y$.”

Although two-fifths of the prospective teachers gave accurate answers to the question that asked them to select functions from the given alternatives and explain their answers, almost none of them offered any explanations for their answers. In addition, nearly 10% of the participants gave incomplete answers to the procedural question about functions, and a quarter of them gave inaccurate answers to it. In addition, more than one-third of them did not answer it. All these things suggest that they had difficulty describing functions. This finding is supported by other studies in the literature. For example, it was reported that students have difficulty understanding the concept of function, although it is one of the most significant mathematical concepts and one of the most important components of mathematical thinking (Gagatsis & Shiakalli, 2004; Karataş & Güven, 2003; Harrell, 2001; O’callaghan, 1998). Similarly, Aydin and Kogce (2008) conducted a study with prospective elementary, secondary and high school teachers and discovered that they were not able to describe the concept of function and failed to offer a formal definition of it. In his study on students’ concept images of the concept of function, Vinner (1983) reported that they considered it to be “a rule of matching,” “an algebraic expression,” “a formula,” “an equation or equality,” “the symbol for y = f (x)” and “schematic expressions in which members of two sets are matched with one another using arrows.” Using Vinner’s (1983) model of concept definition and concept image, Vinner and Dreyfus (1989) examined the concept images of university students and high school students about the concept of function. According to their results, university students considered a function to be “any match between two sets in which the members of the first set are matched with single members of the second,” “dependence between two variables, “a rule,” “an operation,” “a formula” and “an algebraic equation or equality.”

To conclude, the prospective teachers were unable to describe the basic mathematics concepts in this study and failed to offer a formal definition of them. Therefore, memorization-based approaches should be avoided when these concepts are taught, especially in secondary school and high school. Instead, teachers should use materials such as concept maps and concept networks, which are more likely to contribute to permanent and interconnected learning. While solving the problems, some prospective teachers reported that they were unable to remember the relevant formula and thus either solved the problem inaccurately or skipped the question. Apparently, they wrongly assumed that mathematical concepts are merely formulas to be memorized. Therefore, teachers should adopt a holistic approach to teaching these concepts, place emphasis on definitions, give up on memorization-based approaches and focus on the connections between concepts.

References

Akdeniz, F., Ünlü, Y. & Dönmez, D. (2012). Analize giriş cilt-1, 8. Baskı, Nobel Kitabevi, Adana.

Akkoç, H. (2008). Pre-service mathematics teachers' concept images of radian, International Journal of Mathematical Education in ScienceandTechnology, 39 (7), 857 – 878.

Akkoç, H., (2005). Fonksiyon kavramının anlaşılması: tanımsal özellikler ve çoğul temsiller, Eğitim Araştırmaları, (20), 14-24.

Akpınar, A. (2004). Lise matematik 1, Okay Yayıncılık, Ankara.

Aksu, M. (1997). Student performance in dealing with fraction, The Journal of Educational Research, 90(6), 375-380.

Aktaş, D.Y. (2005). İşbirliğine dayalı grup çalışması ile öğrencilerin geometri anlama düzeylerinin incelenmesi (Yüksek lisans tezi, Karadeniz Teknik Üniversitesi, Trabzon). https://tez.yok.gov.tr/UlusalTezMerkezi adresinden edinilmiştir.

Akuysal, N. (2007). İlköğretim 7. sınıf öğrencilerinin 7. sınıf ünitelerindeki geometrik kavramlardaki yanılgıları (Yüksek lisans tezi, Selçuk Üniversitesi, Konya). https://tez.yok.gov.tr/UlusalTezMerkezi adresinden edinilmiştir.

Akyüz, G. & Hangül, T. (2014). A Study on over coming misconceptions of 6th graders about equations, Journal of Theoretical Educational Science, 7(1), 16-43.

Altun, M. (2010). İlköğretim 2. kademede (6.7.8. sınıflarda) matematik öğretimi(7. baskı), Ankara: Alfa Aktüel Yayınevi.

Ankara.

Aydın, M. & Köğce, D. (2008). Öğretmen adaylarının "denklem ve fonksiyon" kavramlarına ilişkin algıları, Yüzüncü Yıl Üniversitesi, Eğitim Fakültesi Dergisi, 5(1), 46-58.

Baki, A. & Aydın-Güç, F. (2014). Dokuzuncu sınıf öğrencilerinin devirli ondalık gösterimle ilgili kavram yanılgıları, Turkish Journal of Computer and Mathematics Education, 5(2), 176-206.

Baki, A. &Kartal, T. (2002). Lise öğrencilerinin cebir bilgilerinin kavramsal ve işlemsel bilgi bağlamında değerlendirilmesi, V. Ulusal Fen Bilimleri ve Matematik Eğitimi Kongresi, ODTÜ Kültür ve Kongre Merkezi, Ankara.

Balcı, M. (2012). Matematik analiz-1, 8. Baskı, Sürat Üniversite Yayınları, İstanbul.

Baştürk, S. (2009). Mutlak değer kavramı örneğinde öğretmen adaylarının öğrenci hatalarına yaklaşımları. Necatibey Eğitim Fakültesi Elektronik Fen ve Matematik Eğitimi Dergisi, 3(1),174–194.

Baykul, Y. (1999). İlköğretimde Matematik Öğretimi, Öğretmen El Kitabı: Modül 6, Milli Eğitim Yayınları.

Baykul, Y. (2009). İlköğretimde matematik öğretimi (6-8. sınıflar), (1. baskı). Ankara: Pegem Akademi Yayınevi.

Behr, M. J., Harel, G., Post, T. R., & Lesh, R. (1992). Rational number, ratio, and proportion, In D. A. Grouws (Ed.), Handbook of research on mathematics teaching and learning, New York: Macmillan.

Bingolbali, E. & Monaghan, J. (2008). Concept image revisited, Educational Studies in Mathematics, 68(1),19-35.

Bingölbali, E. & Özmantar, M. F. (2010). İlköğretimde karşılaşılan matematiksel zorluklar ve çözüm önerileri (2. baskı),Pegem Akademi Yayınevi, Ankara.

Birgin, O. & Gürbüz, R. (2009). İlköğretim II. kademe öğrencilerinin rasyonel sayılar konusundaki işlemsel ve kavramsal bilgi düzeylerinin incelenmesi, Uludağ Üniversitesi Eğitim Fakültesi Dergisi, 22(2), 529-550.

Bozkurt, A. & Koç, Y. (2012). Investigating first year elementary mathematics teacher education students' knowledge of prism, EducationalSciences: Theory and Practice, 12(4), 2949-2952.

Çepni, S. (2007). Araştırma ve proje çalışmalarına giriş (2. Baskı), Celepler Matbaacılık, Trabzon.

Çiltaş, A., Işık, A. & Kar, T. (2010). The concept of absolute value: evaluation of procedural and conceptual knowledge,Journal of Institute of Mathematics & Computer Science, 21(1), 131-139.

De Villiers, M. (1994). The role and function of a hierarchical classisication of quadrilaterals, For The Learning of Mathematics, 14(1), 11-18.

Demirtaş, A., (1986), Ansiklopedik matematik sözlüğü, Bilim Teknik Kültür Yayınları, Ankara.

Dernek, A. (2009). Genel matematik, 4. Basım, Nobel Akademik Yayıncılık, Ankara.

Dönmez, A., (2002). Matematiğin öyküsü ve serüveni, Toplumsal Dönüşüm Yayınları, İstanbul.

Duatepe-Paksu, A., Musan, M., İymen, E. & Pakmak, G.S. (2012). Sınıf öğretmeni adaylarının boyut konusundaki kavram görüntüleri, Buca Eğitim Fakültesi Dergisi, 34, 53-68.

Duffy, T.,Mottershead, L. &Murty, G. (2003). Connections maths 8. Pascal Press, Sydney, Australia.

Durmuş, S. (2005). Identifying pre-service elementary school teachers' conceptualization levels of rational numbers, EducationalSciences: Theory & Practice, 5(2), 639-666.

Duru, A. (2011). Pre-service teachers' perception about the concept of limit, Educational Sciences: Theory and Practice, 11(3), 1710-1715.

EBA, (2015). 9-12. Sınıf etkileşimli matematik sözlüğü, http/f.eba.gov.tr/matsozluk/ sozluk.html, adresinden erişilmiştir.

Ekiz, D. (2003). Eğitimde araştırma yöntem ve metotlarına giriş: nitel, nicel ve eleştirel kuram metodolojileri, Anı Yayıncılık, Ankara.

Ergün, S. (2010). İlköğretim 7. sınıf öğrencilerinin çokgenleri algılama, tanımlama ve sınıflama biçimleri (Yüksek lisans tezi, Dokuz Eylül Üniversitesi, İzmir). https://tez.yok.gov.tr/UlusalTezMerkezi adresinden edinilmiştir.

Erşen-Bahar, Z. & Karakuş, F. (2013). Evaluation of preservice elementary teachers' concept images for quadrilaterals, TurkishJournal of Computer and Mathematics Education, 4(2), 124-146.

Fischbein, E., Jehiam, R. & Cohen, D. (1995). The concept of irrasyonel numbers in high school students and prospective teachers, EducationalStudies in Mathematics, 29(1), 29-44.

Fujita, T. & Jones, K. (2007). Learners' understanding of the definitions and hierarchical classification of quadrilaterals: Towards a theoretical framing, Research in Mathematics Education, 9 (1-2), 3-20.

Gagatsis, A. ve Shiakalli, M., (2004), Ability to Translate From One Representation of The Concept of Function to Another and Mathematical Problem Solving, Educational Psychology, 24(5), 645–657.

Gronlund, N. E. &Linn, R. L. (1990). Measurement and evaluation in teaching (6th ed.), New York, NY: MacMillan.

Harrell, G.K., (2001), The Effects of Two Technologies on College Algebra Students' Understanding of The Concept of Function, Unpublished Dissertation, Universtyof Florida.

İpek, A.S., Işık, C. & Albayrak, M. (2005). Sınıf öğretmeni adaylarının kesir işlemleri konusundaki kavramsal becerileri, Kazım Karabekir Eğitim Fakültesi Dergisi, 1, 537-547.

İşleyen, T. & Işık, A. (2005). Alt vektör uzayı kavramının kavramsal öğrenilmesi üzerine, Kazım Karabekir Eğitim Fakültesi Dergisi, 11, 493-501.

Kadıoğlu, E. &Kamali, M. (2013). Genel Matematik, Kültür Eğitim Vakfı Yayınevi, Erzurum.

Kaplan, A., İşleyen, T. & Öztürk, M. (2011). 6. sınıf oran orantı konusundaki kavram yanılgıları, Kastamonu Eğitim Dergisi, 19(3), 953-968.

Kara, F. & Delice, A. (2012). Kavram tanımı mı? Yoksa kavram imgeleri mi? İrrasyonel sayıların temsilleri, X. Ulusal Fen Bilimleri ve Matematik Eğitimi Kongresi. Niğde, Türkiye

Karataş, İ. ve Güven, B., (2003), Fonksiyon kavramının farklı öğrenim düzeyinde olan öğrencilerdeki gelişimi, Eurasian Journal of educational Research, 4(16), 64-73.

Komisyon, (2014). Ortaöğretim matematik 9. sınıf 1. kitap, Milli Eğitim Bakanlığı Yayınları(5890).

Lima, R.N. &Tall, D. (2006). What does equation mean? A brainstorm of the concept, Third International Conference on the Teaching of Mathematics, Istanbul.

Mack, N. (1995). Confounding whole-number and fraction concept when building on informal knowledge, Journal for Research Mathematics Education, 26(5), 422-441.

Merriam, S. B. (1988). Case study research in education: A qualitative approach,CA: Jossey-Bass, San Francisco.

Miles, M. &Huberman, M. (1994). An expanded source book qualitative data analysis, (2th edition), America: Person Education.

Milli Eğitim Bakanlığı(MEB), (2012). Ortaöğretim matematik 9. sınıf ders kitabı. Devlet Kitapları: İstanbul.

Monaghan, F. (2000). What difference does it make? Children's views of the differences between some quadrilaterals, Educational Studies in Mathematics, 42(2),179-196.

Moralı, S.,Köroğlu, H. & Çelik, A.(2004). Buca eğitim fakültesi matematik öğretmen adaylarının soyut matematik dersine yönelik tutumları ve rastlanan kavram yanılgıları, Gazi Eğitim Fakültesi Dergisi, 24(1), 161-175.

Moss, J. & Case, R. (1999). Developing children's understanding of the rational numbers: A new model and an experimental curriculum, Journal for Research in Mathematics Education, 30, 122-147.

O'callaghan, B., (1998), Computer-Intensive Algebra And Students'conceptual Knowledge of Functions, Journal For Research In Mathematics Education,(29), 21-40.

Oaks, A.B., (1990). Writing to learn mathematics: Why do we need it and how can it help us?,Associations of Mathematics Teachers of New York States Conference, Ellenvile.

Özkaya, M. & İşleyen, T. (2012). Fonksiyonlarla ilgili bazı kavram yanılgıları, Çankırı Karatekin Üniversitesi Sosyal Bilimler Enstitüsü Dergisi, 3(1), 01-32.

Polatoğlu, M., Çamlı, A. & Çalıkoğlu, İ., (2001). İlköğretim matematik ders kitabı 8, 3. Baskı, Milli Eğitim Basımevi, İstanbul.

Rösken, B. & Rolka, K. (2007). Integrating intuition: the role of concept image and concept definition for students' learning of integral calculus, The Montana Mathematics Enthusiast, 3, 181-204.

Simon, M. (1993). Prospective elementary teachers' knowledge of division, Journal for Research in Mathematics Education, 24, 233-254.

Soylu, Y. & Aydın, S. (2006). Matematik derslerinde kavramsal ve işlemsel öğrenmenin dengelenmesinin önemi üzerine bir çalışma, Erzincan Eğitim Fakültesi Dergisi, 8(2), 83-95.

Soylu, Y., Akgün, L., Dündar, S. & İşleyen, T. (2011). Indicating that one number is an irrational number with a different method, Procedia Social and Behavioral Sciences,15, 3277–3280.

Tall, D. &Vinner, S. (1981). Concept image and concept definition in mathematics with particular reference to limits and continuity, EducationalStudies in Mathematics, 12(2), 151-16.

Toluk, Z. (2002). İlkokul öğrencilerinin bölme işlemi ve rasyonel sayıları ilişkilendirme süreçleri, Boğaziçi Üniversitesi Eğitim Dergisi, 19 (2), 81-103.

Topçu, T., Kertil, M., Akkoç, H., Yılmaz, K. & Önder, O. (2006). Pre-service and in-service mathematics teachers' concept images of radian, 30 th International Conference on the Psychology of Mathematics Education, Prague, Czech Republic.

Tortumlu, F. & Kılıç, A., (2000). İlköğretim matematik ders kitabı, Milli Eğitim Basımevi, İstanbul.

Türnüklü, E., Gündoğdu-Alaylı, F. & Akkaş, E. N. (2013). Investigation of prospective primary mathematics teachers' perceptions and images for quadrilaterals, EducationalSciences: Theory&Practice, 13(2), 1225-1232.

Vinner, S. (1983). Concept definition concept image and the notion of function,International Journal for Mathematics Education in Science and Technology, 14(3), 293–305.

Vinner, S. (1991). The role of definitions in the teaching and learning of mathematics. In D. Tall (Eds.). Advanced Mathematical Thinking. Dordrecht: KluwerAcademic.

Vinner, S.,& Dreyfus, T. (1989). Images and definitions for the concept of function,Journal for Research in Mathematics Education, 20(4), 356-366.

Wilson, P.S. (1990). Inconsistent ideas related to definitions and examples, Focus on Learning Problems in Mathematics, 12(3), 31-47.

Yenilmez, K. & Avcu, T. (2009). İlköğretim öğrencilerinin mutlak değer konusunda karşılaştıkları zorluklar, Dicle Üniversitesi Ziya Gökalp Eğitim Fakültesi Dergisi, 12, 80-88.

Yenilmez, K. ve Avcı, T. (2009). İlköğretim öğrencilerinin mutlak değer konusunda karşılaştıkları zorluklar, Dicle Üniversitesi Ziya Gökalp Eğitim Fakültesi Dergisi, 12, 80-88.

Yılmaz, Z. & Yenilmez, K. (2008). İlköğretim 7. ve 8. sınıf öğrencilerinin ondalık sayılar konusundaki kavram yanılgıları (Uşak İli Örneği), Afyon Kocatepe Üniversitesi Fen Bilimleri Dergisi, 8(1), 291-312.

Zembat, İ.Ö.,Özmantar, M.F., Bingölbali, E., Şandır, H.& Delice, A.(Editörler) (2013). Tanımları ve Tarihsel Gelişimiyle Matematiksel Kavramlar, Pegem Akademi, Ankara.